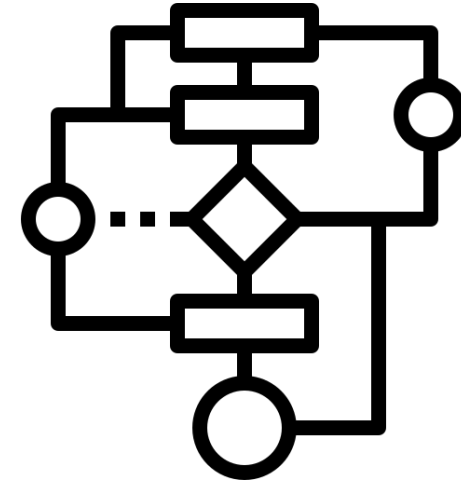
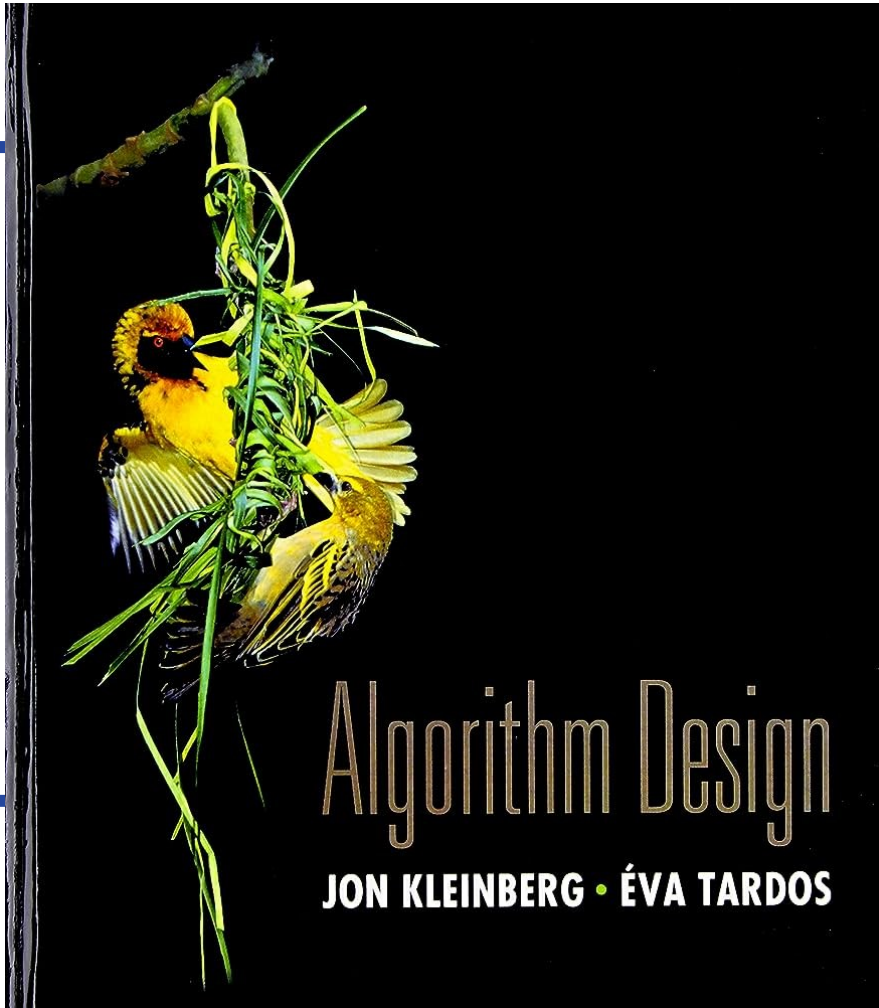




CS 310: Algorithms

Lecture 8

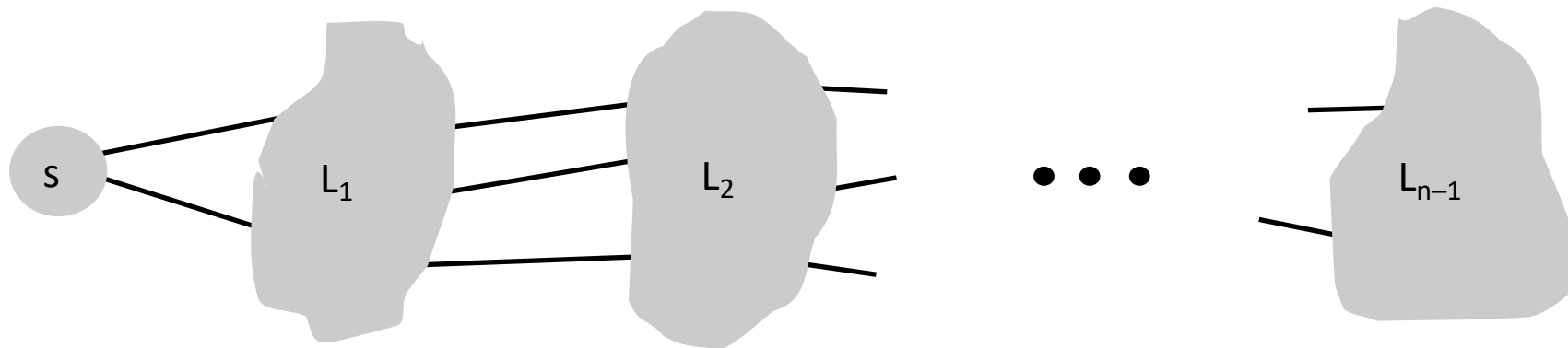
Instructor: Naveed Anwar Bhatti



Chapter 3: Graphs

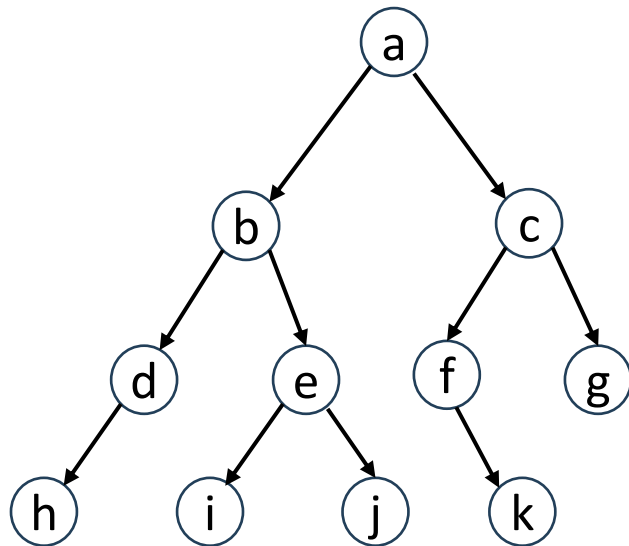
Breadth First Search (BFS)

- BFS traverses the vertices of a graph from a given source node s in the order of the distances from the source
 - Discovers all vertices at distance k from s before discovering any vertices at distance $k + 1$
 - Outputs a BFS tree with root s

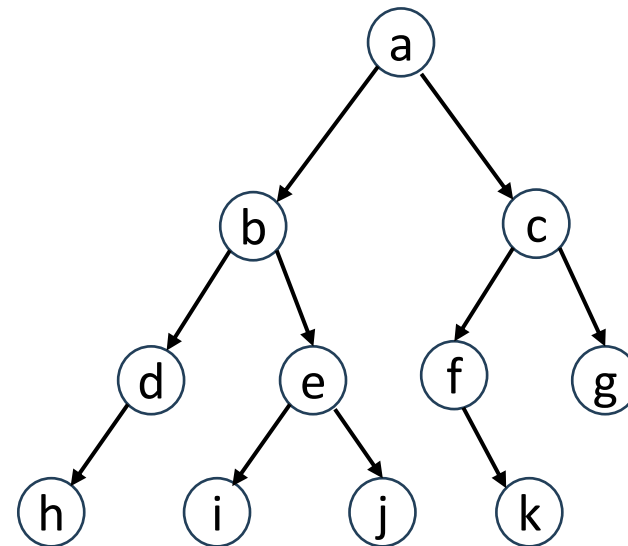


Depth First Search (DFS)

- DFS follows the strategy of going deep first and then wide
 - DFS starts at the source node s and explores as far as possible along each branch before backtracking



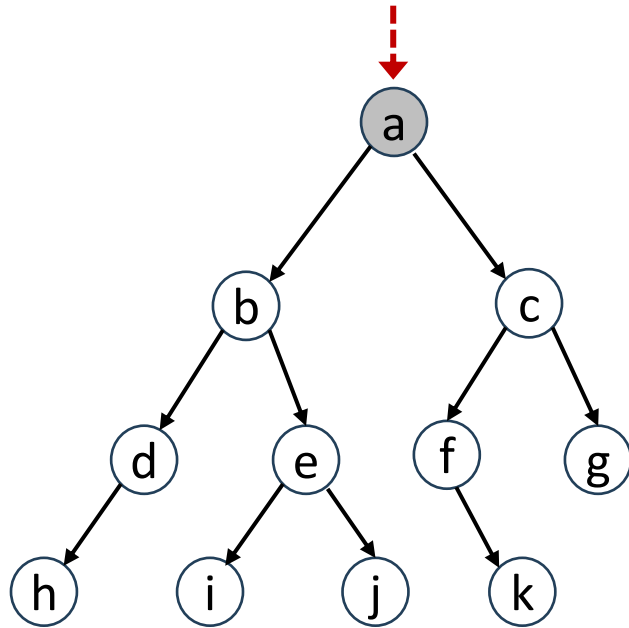
Depth First Search



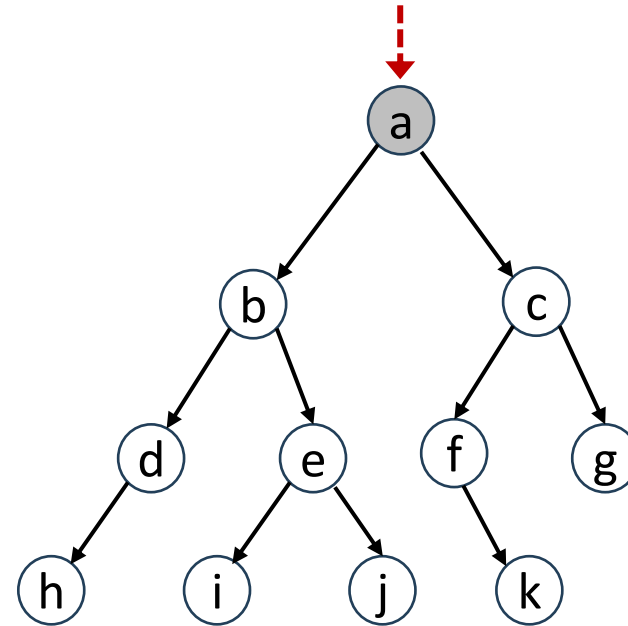
Breath First Search

Depth First Search (DFS)

- DFS follows the strategy of going deep first and then wide
 - DFS starts at the source node s and explores as far as possible along each branch before backtracking



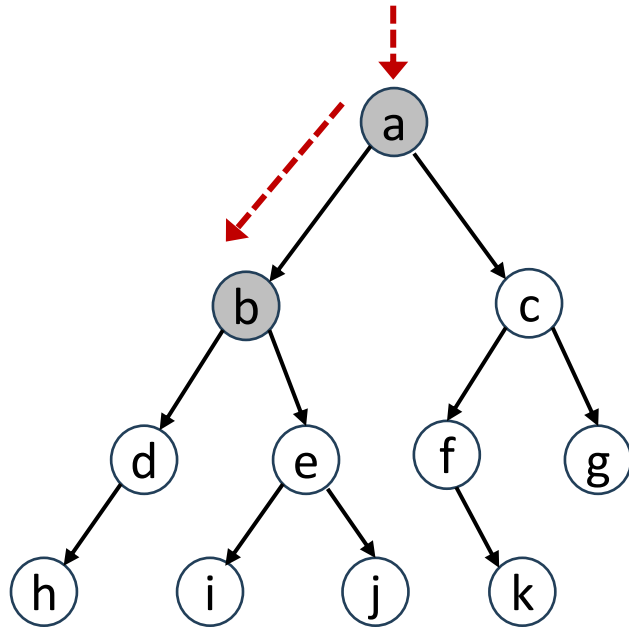
Depth First Search



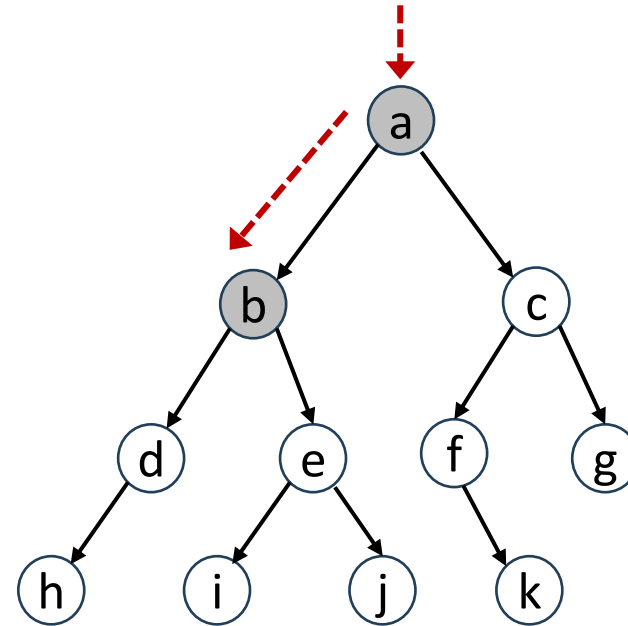
Breadth First Search

Depth First Search (DFS)

- DFS follows the strategy of going deep first and then wide
 - DFS starts at the source node s and explores as far as possible along each branch before backtracking



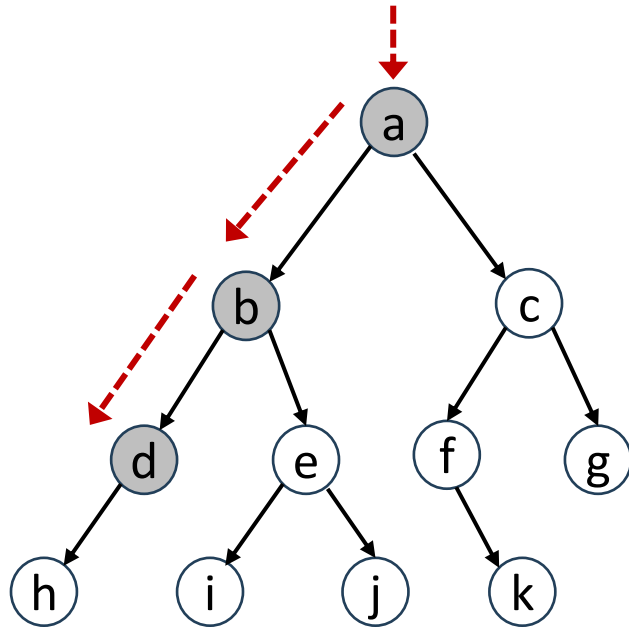
Depth First Search



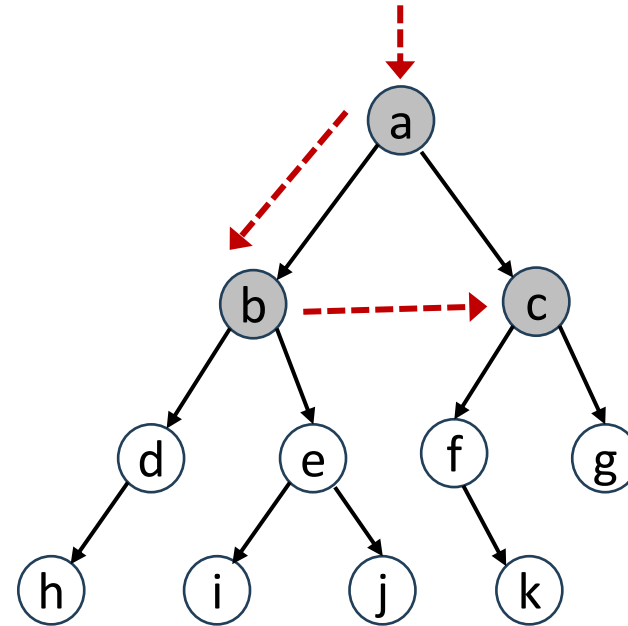
Breadth First Search

Depth First Search (DFS)

- DFS follows the strategy of going deep first and then wide
 - DFS starts at the source node s and explores as far as possible along each branch before backtracking



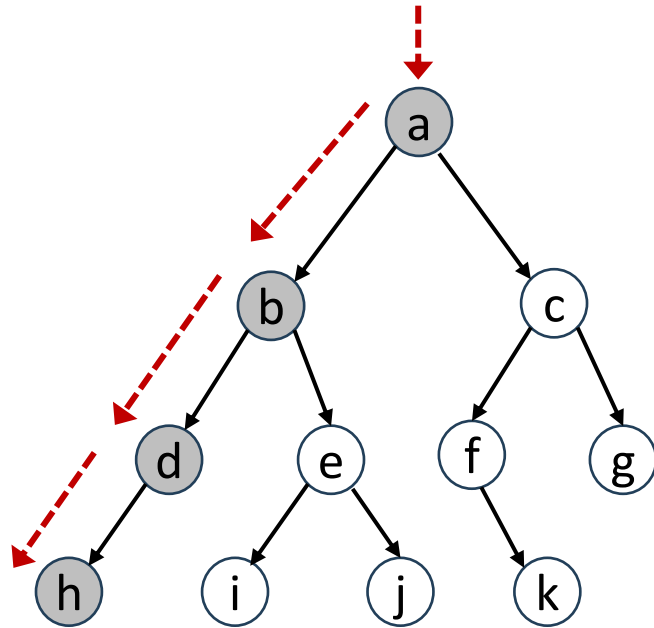
Depth First Search



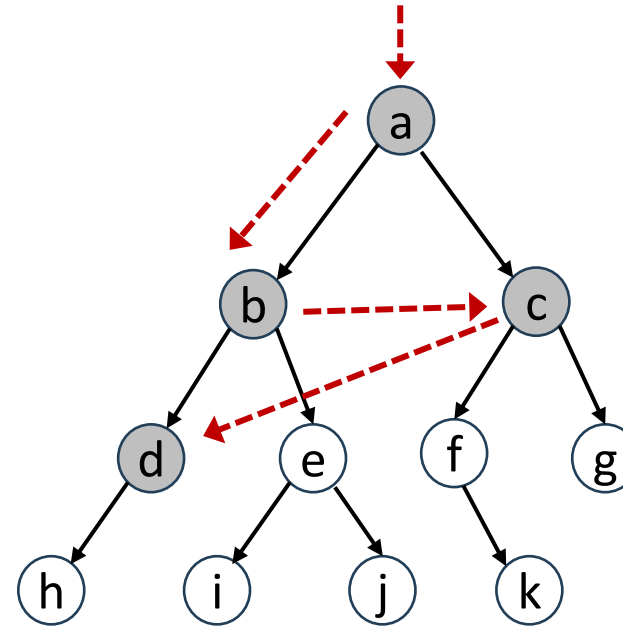
Breadth First Search

Depth First Search (DFS)

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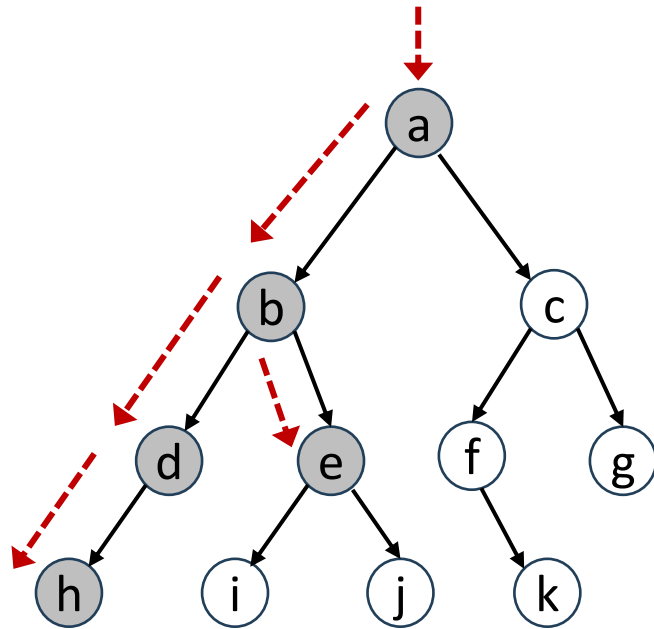
Depth First Search



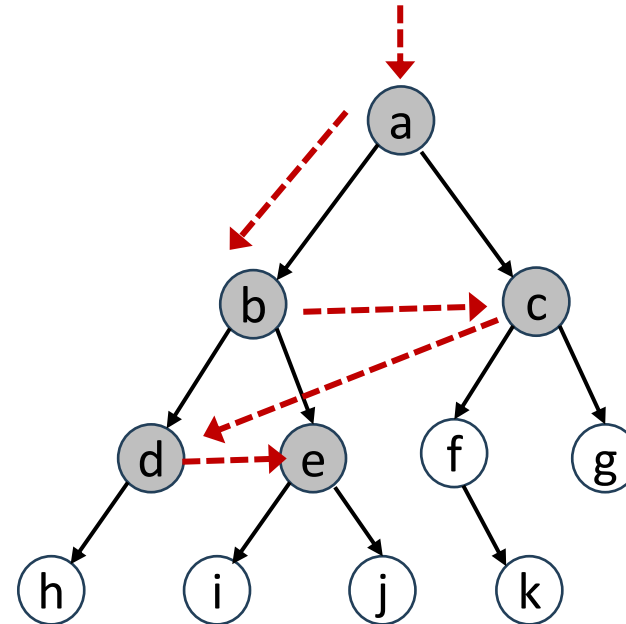
Breadth First Search

Depth First Search (DFS)

- DFS follows the strategy of going deep first and then wide
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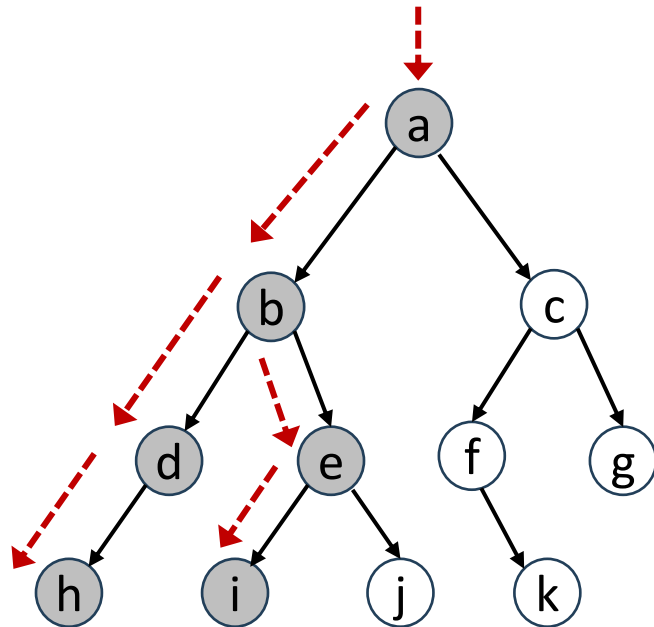
Depth First Search



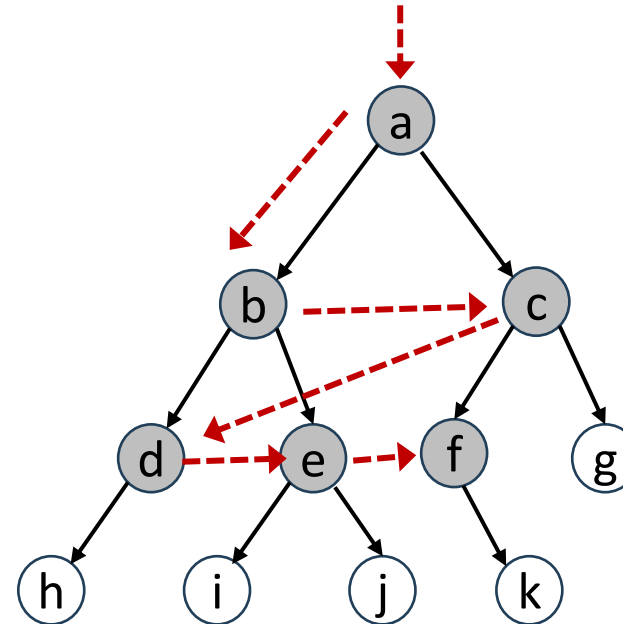
Breadth First Search

Depth First Search (DFS)

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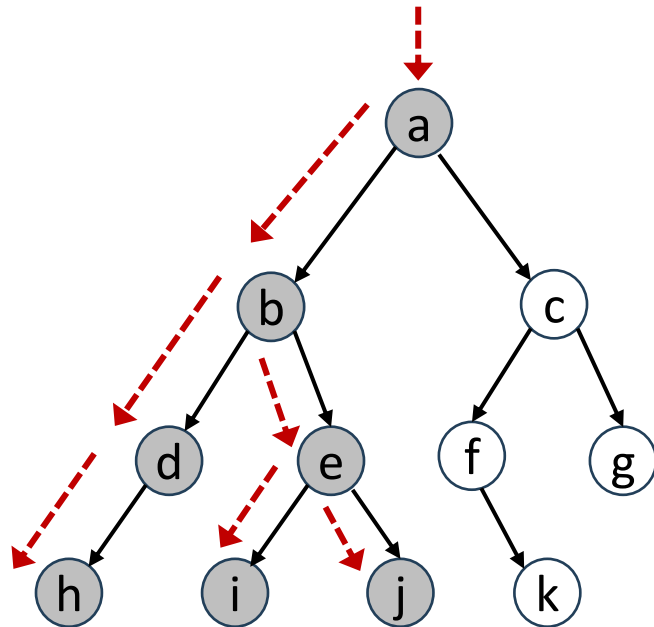
Depth First Search



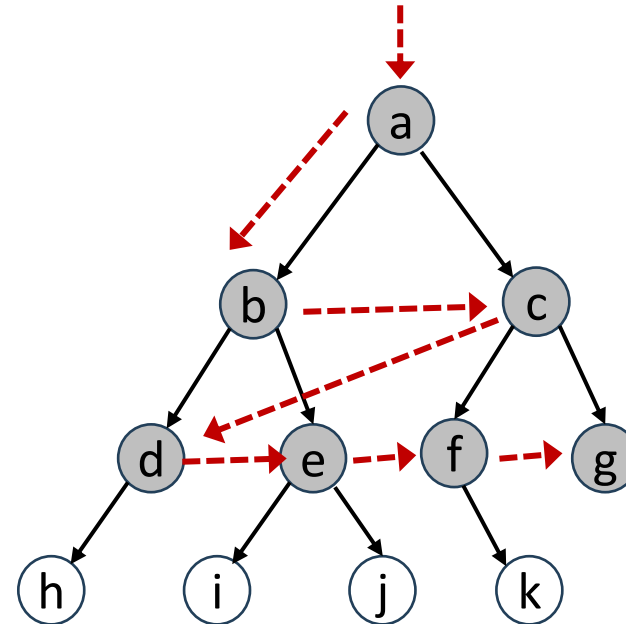
Breadth First Search

Depth First Search (DFS)

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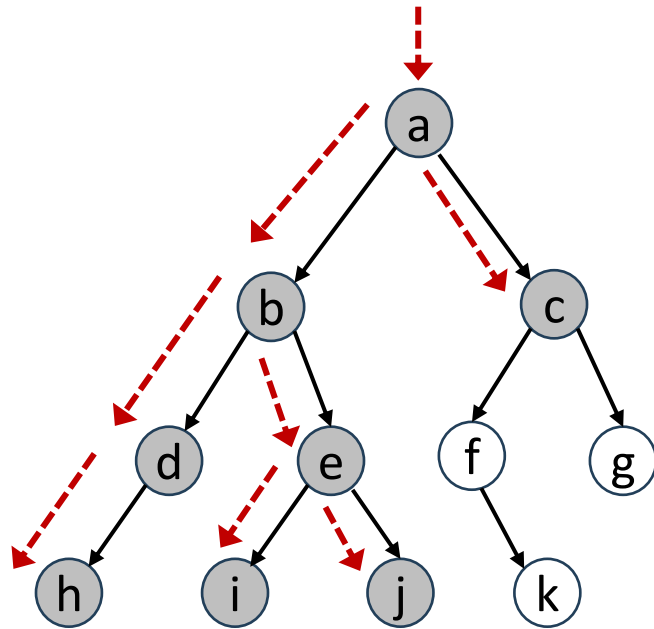
Depth First Search



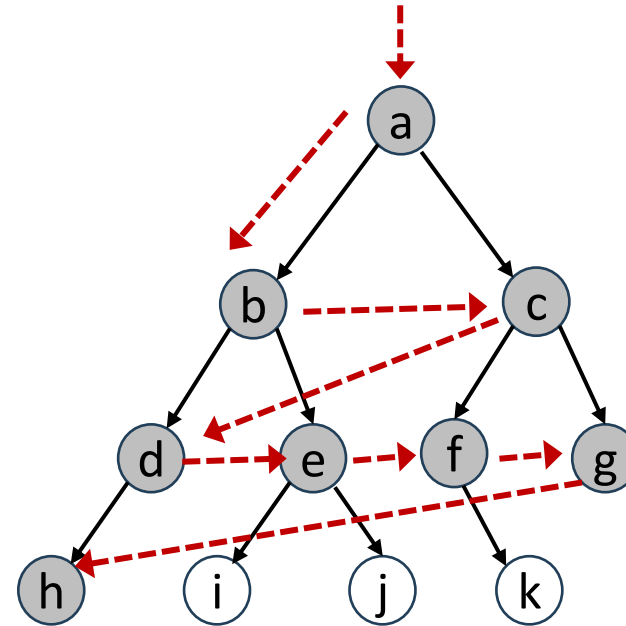
Breadth First Search

Depth First Search (DFS)

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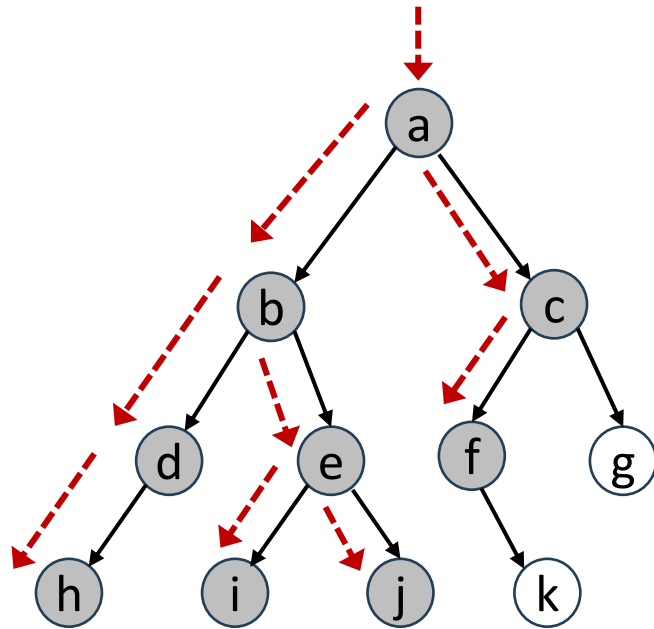
Depth First Search



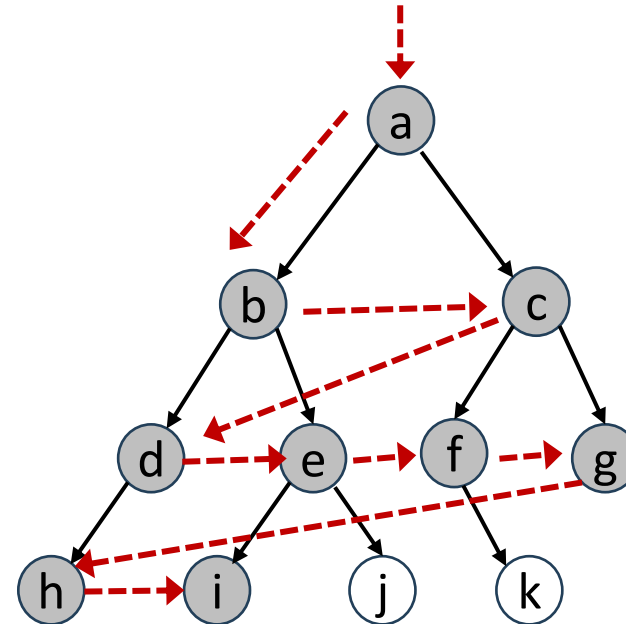
Breadth First Search

Depth First Search (DFS)

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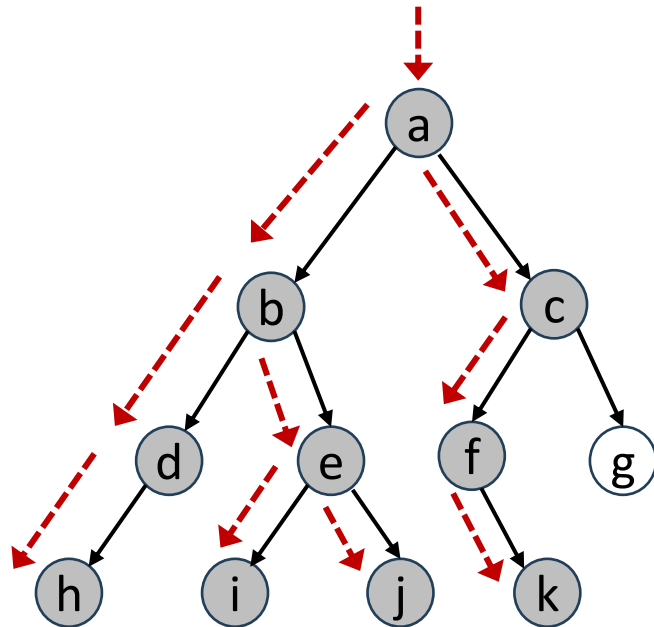
Depth First Search



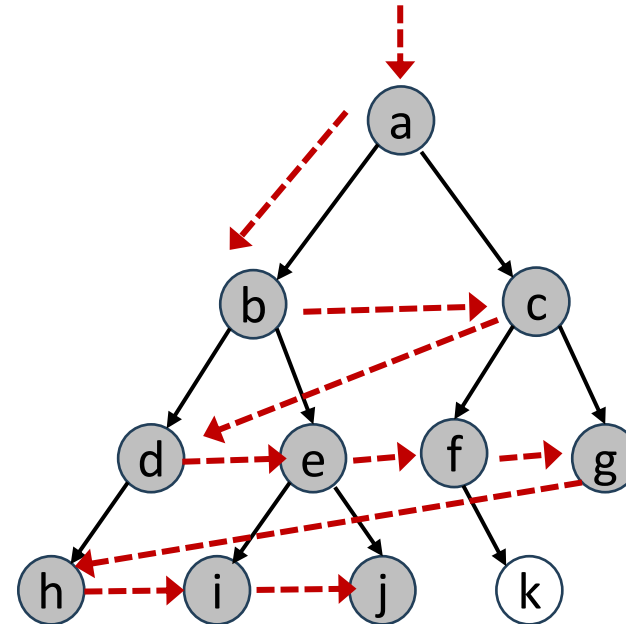
Breadth First Search

Depth First Search (DFS)

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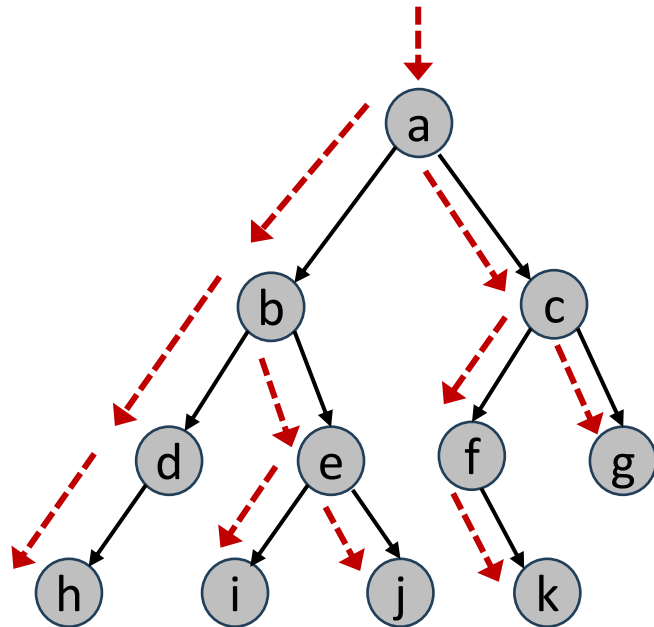
Depth First Search



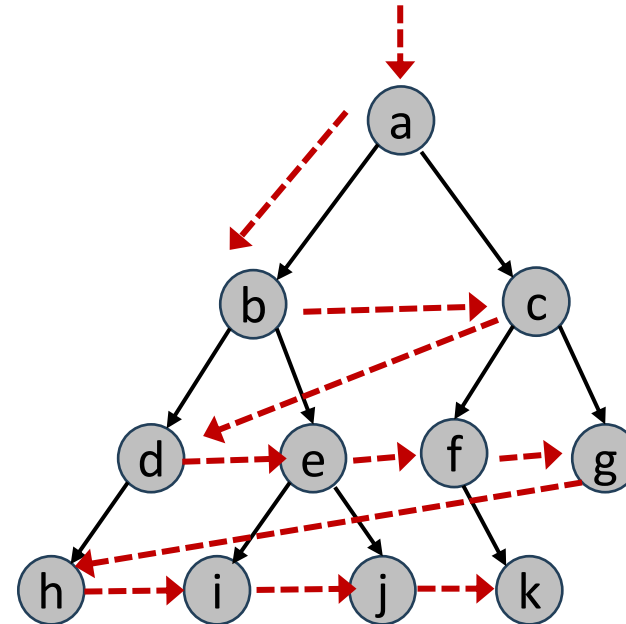
Breadth First Search

Depth First Search (DFS)

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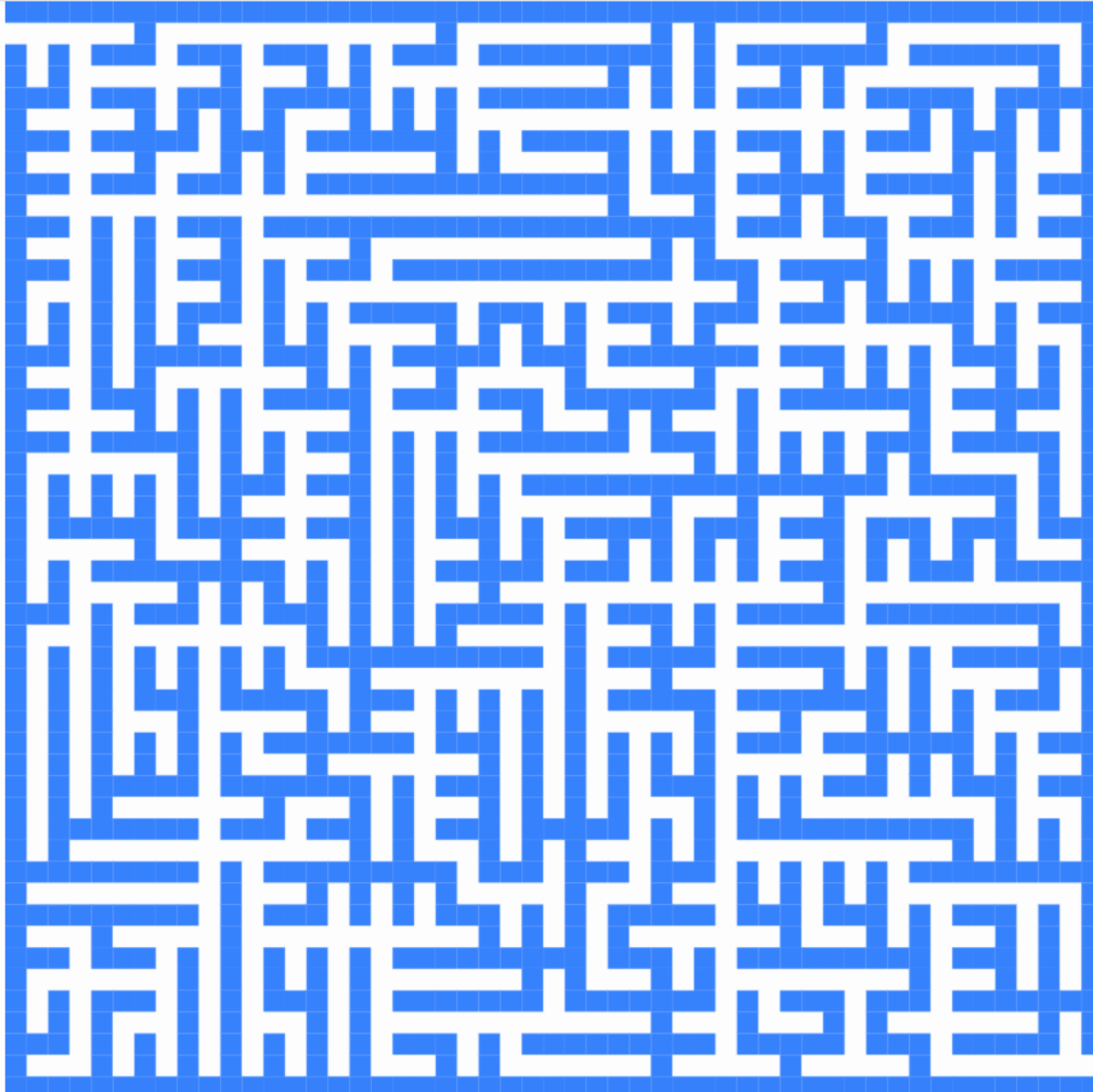
Depth First Search



Breadth First Search

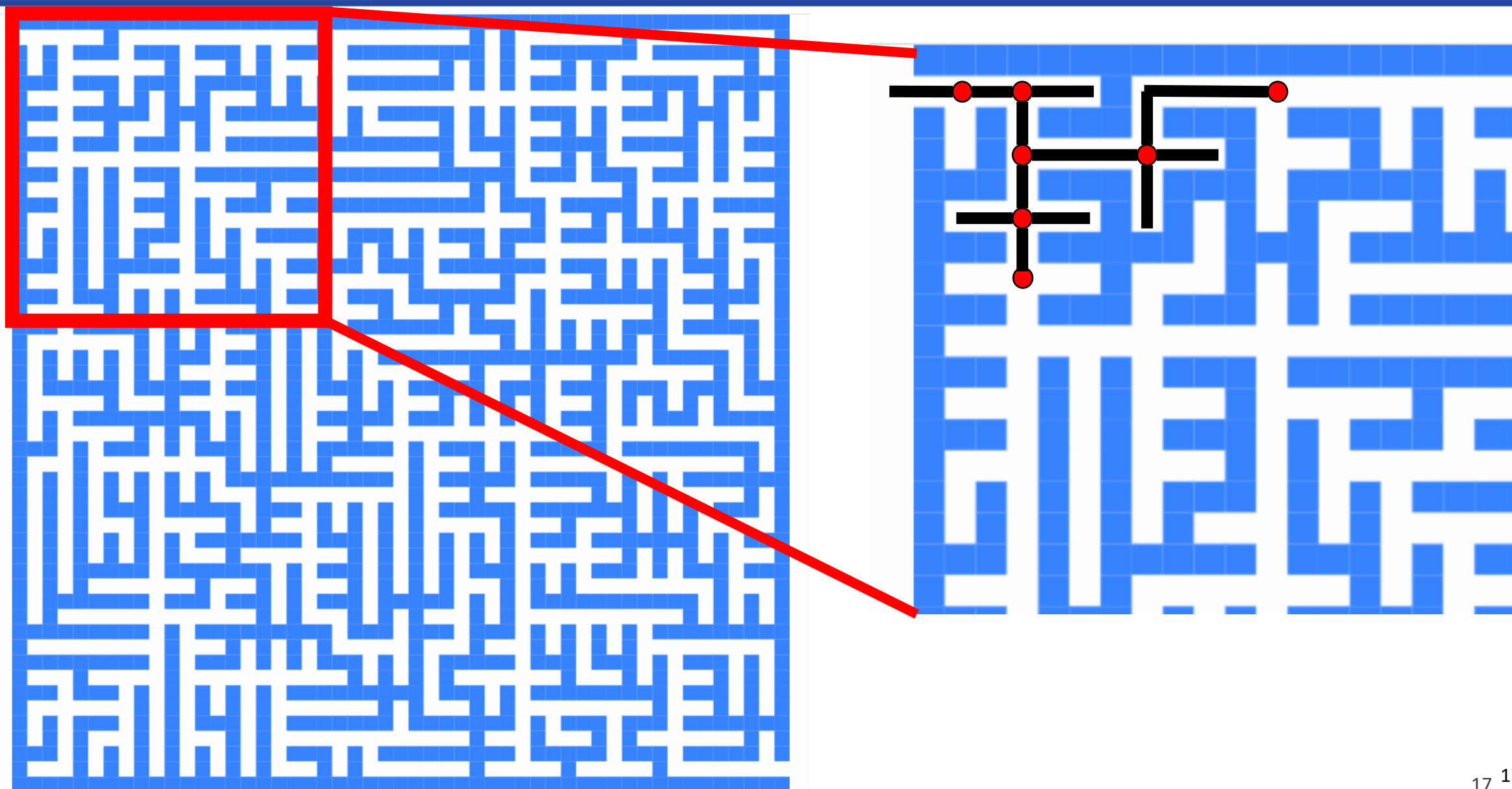
Who can solve this MAZE?

Start



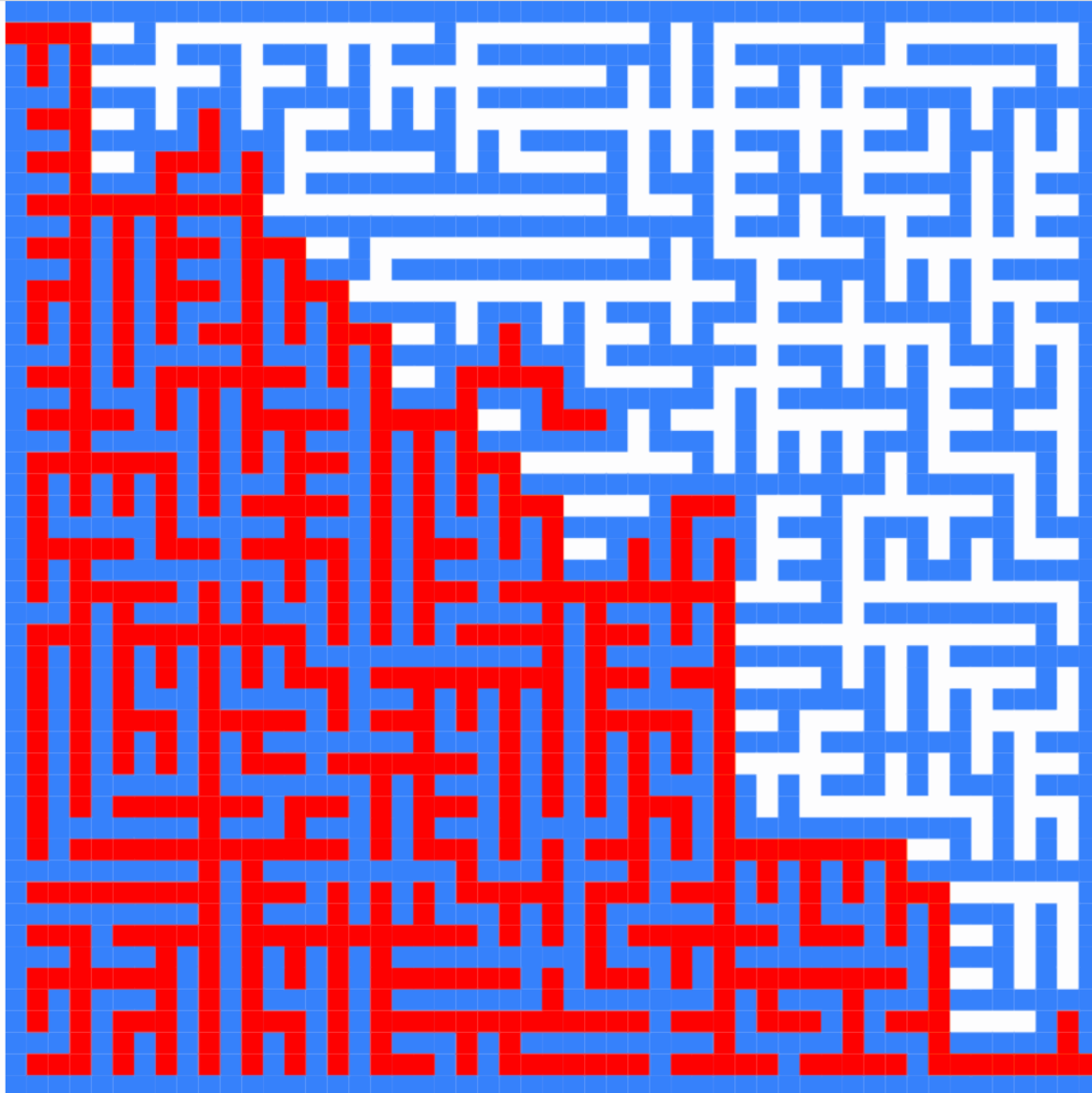
Stop

It's a Tree

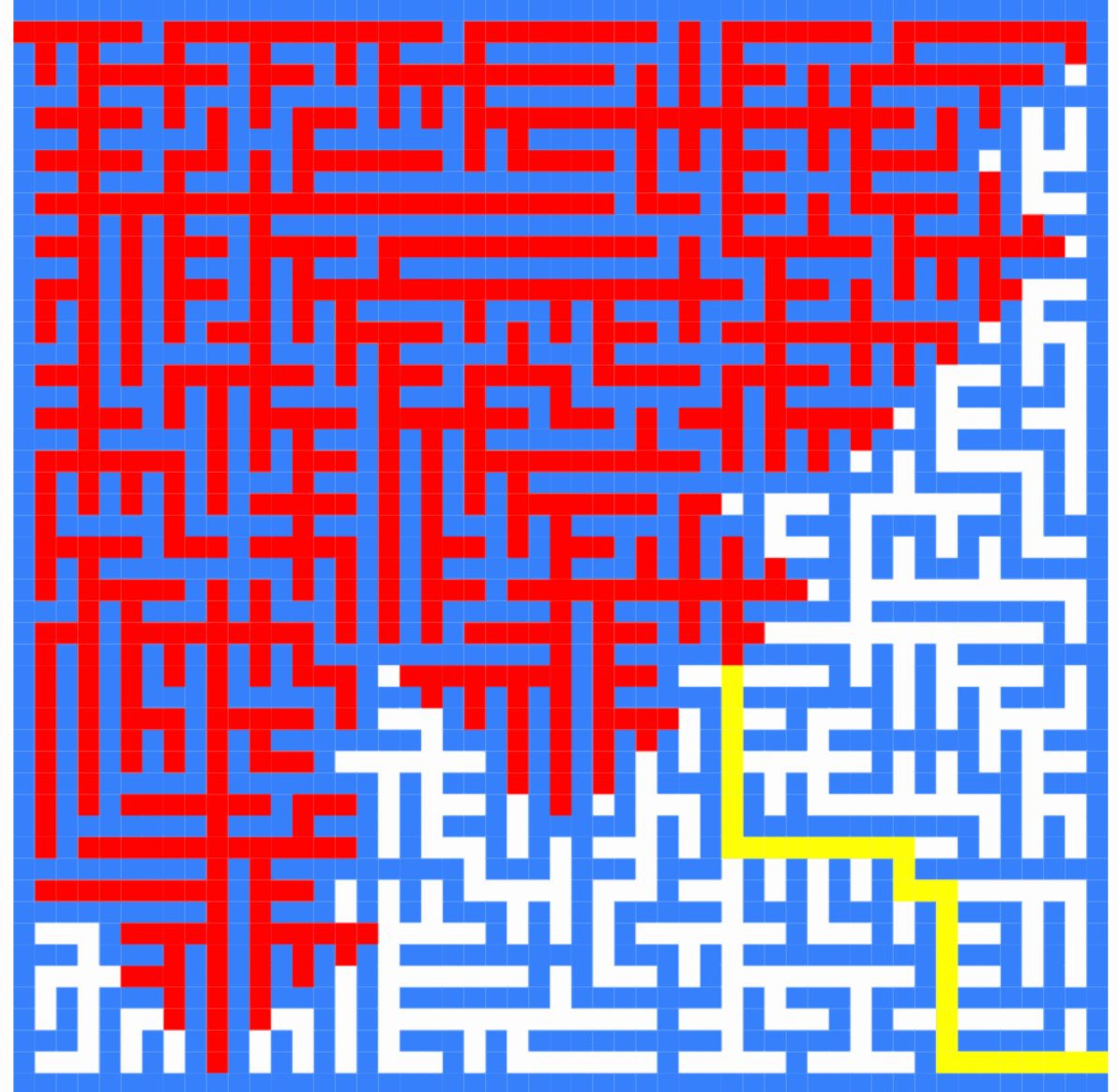




DFS



BFS



Few interesting videos...



Can water solve a maze?

Steve Mould ✓

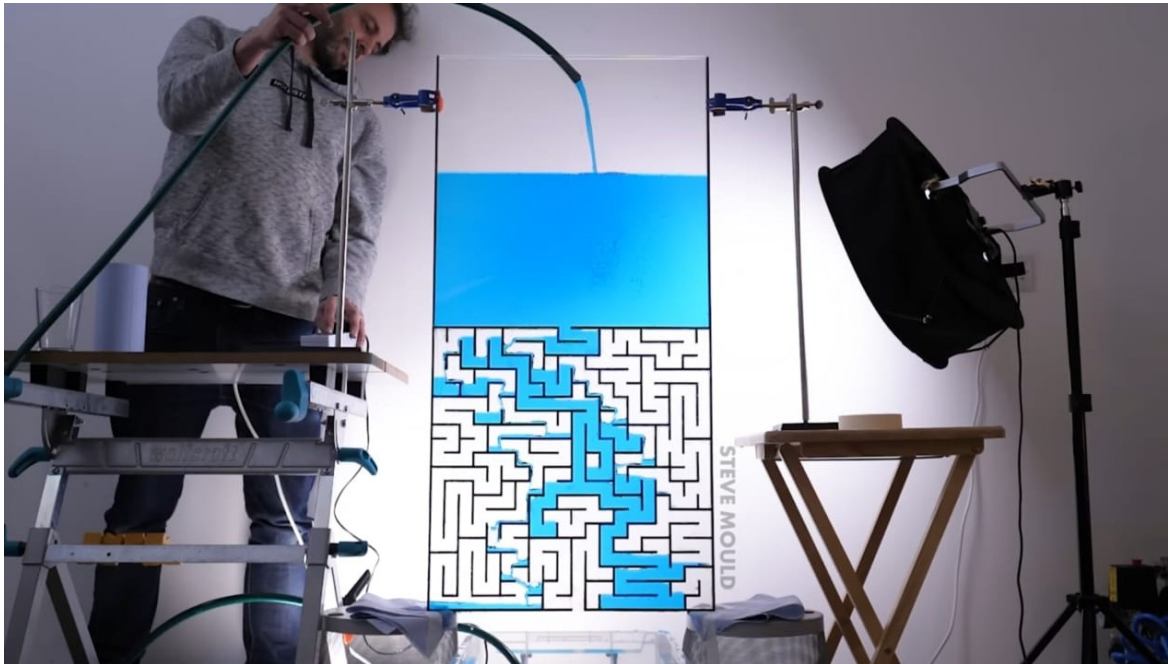
10M views • 5 months ago



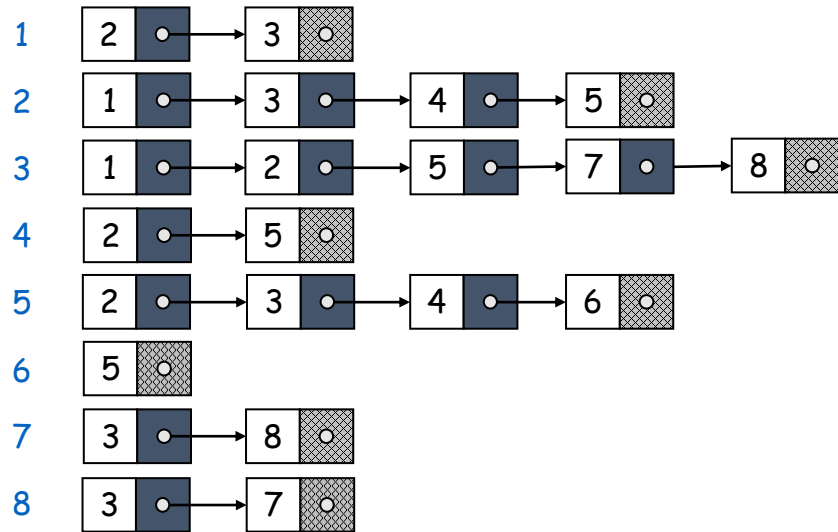
The Fastest Maze-Solving Competition On Earth

Veritasium ✓

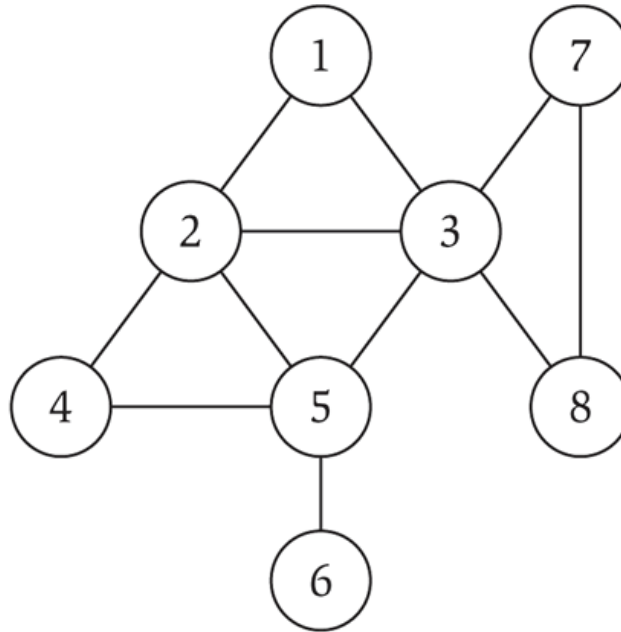
12M views • 4 months ago



Adjacency List

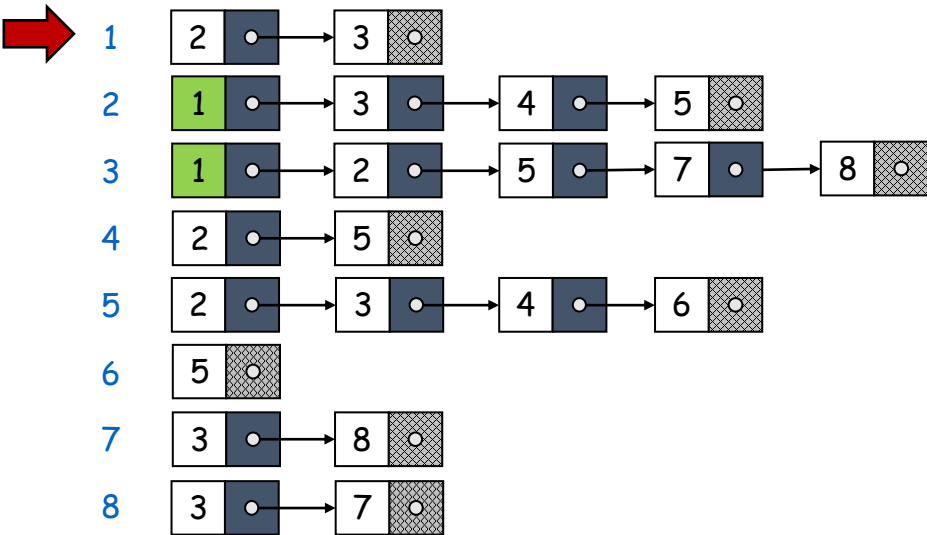


Graph

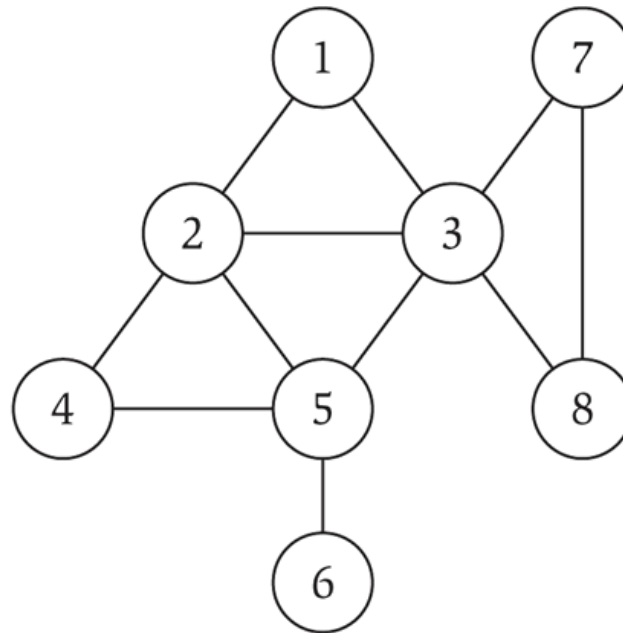


DFS Tree

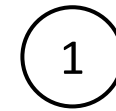
Adjacency List



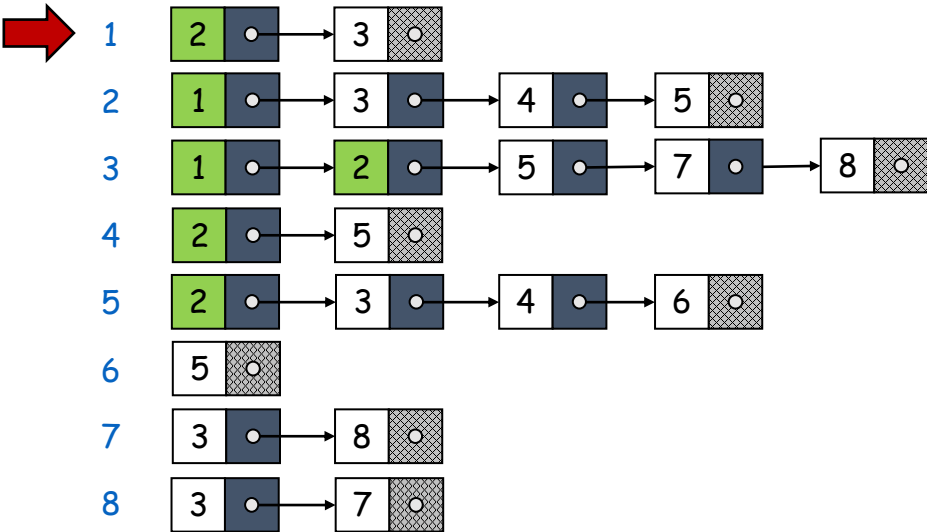
Graph



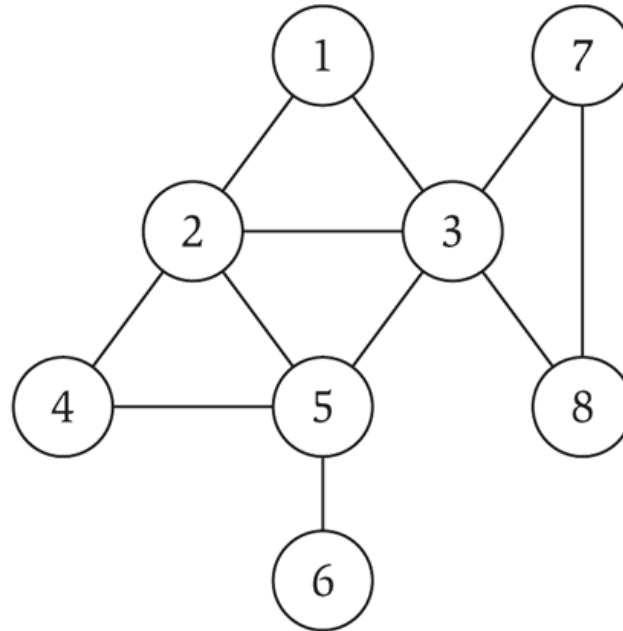
DFS Tree



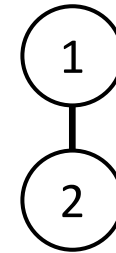
Adjacency List



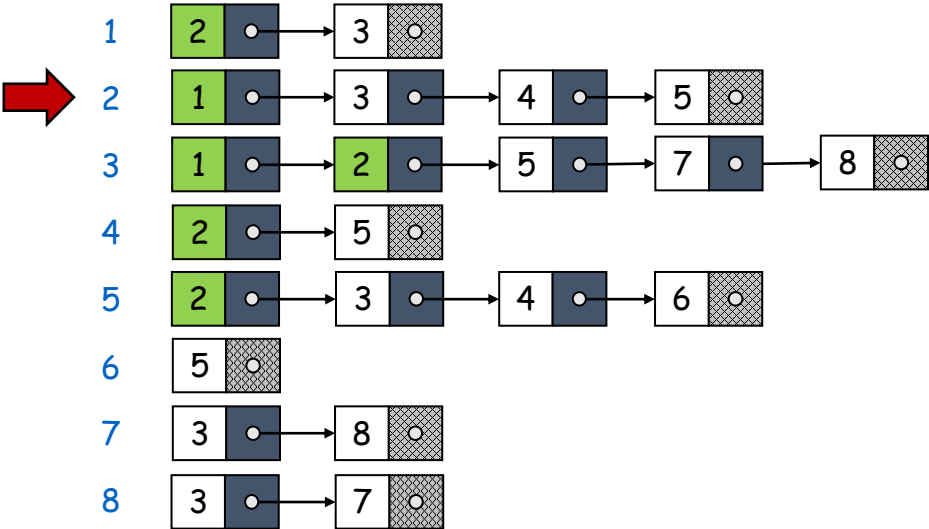
Graph



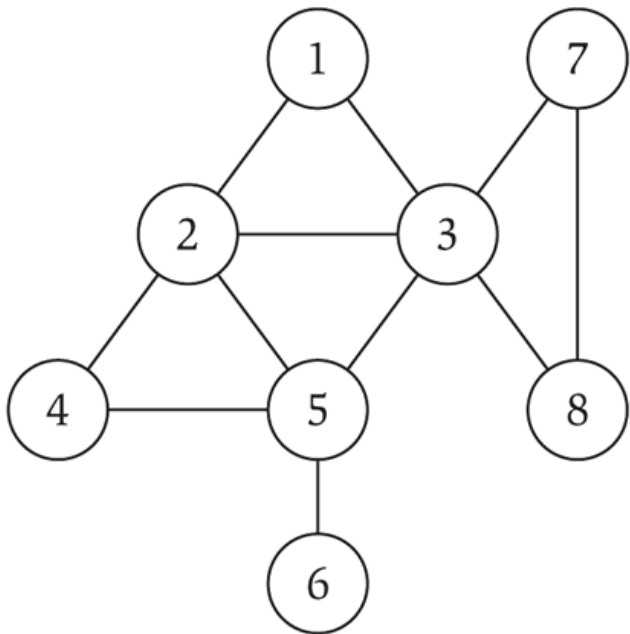
DFS Tree



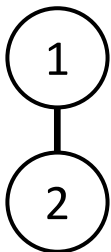
Adjacency List



Graph



DFS Tree



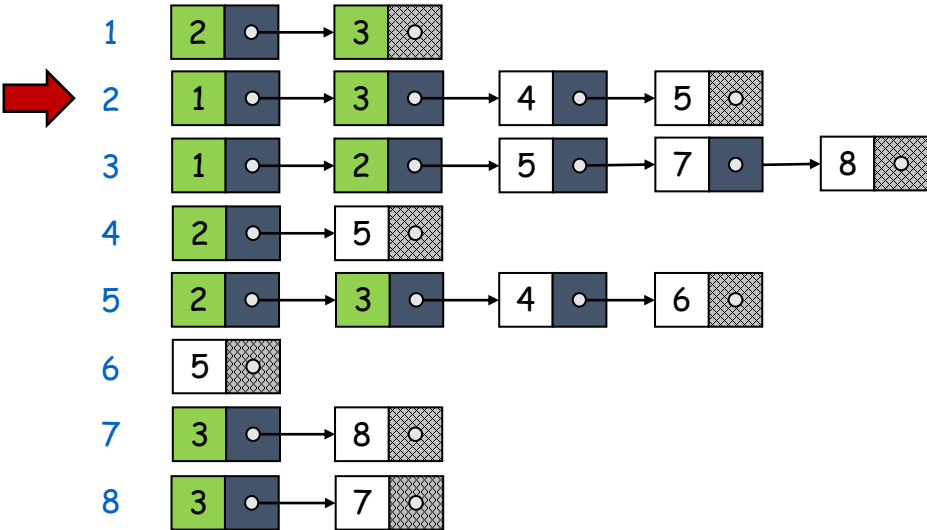
Stack

1

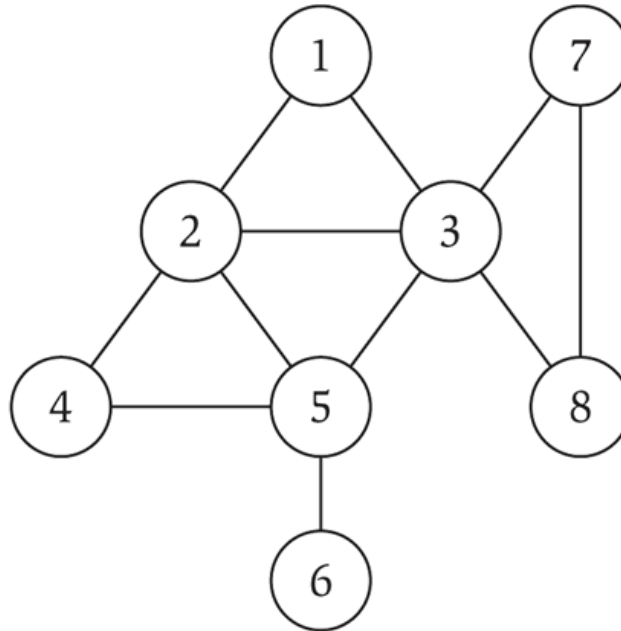
2

3

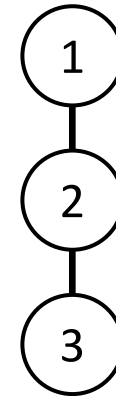
Adjacency List



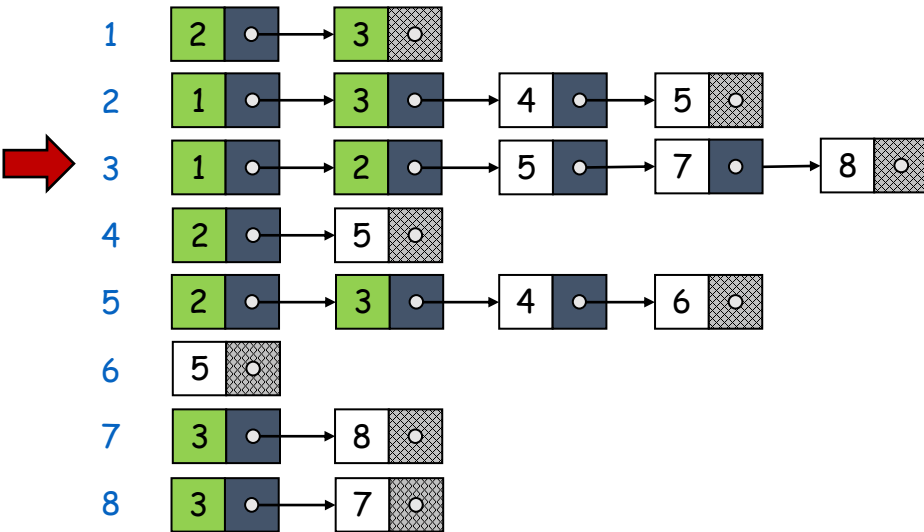
Graph



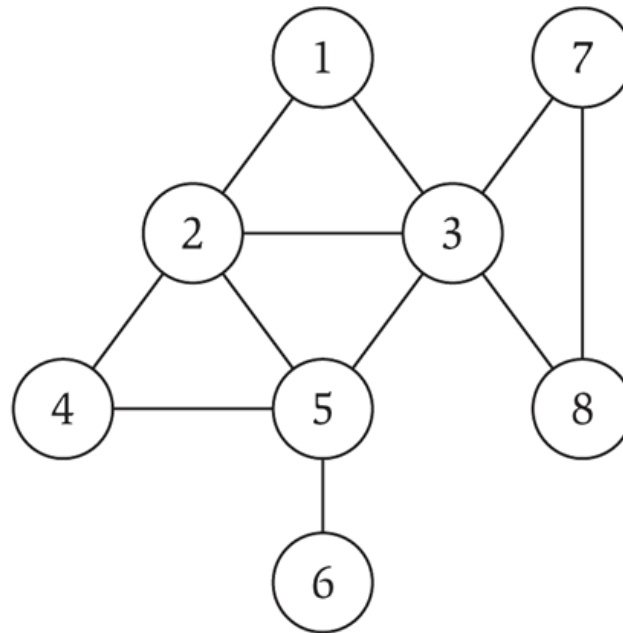
DFS Tree



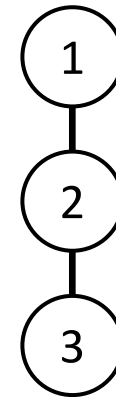
Adjacency List



Graph



DFS Tree



Stack

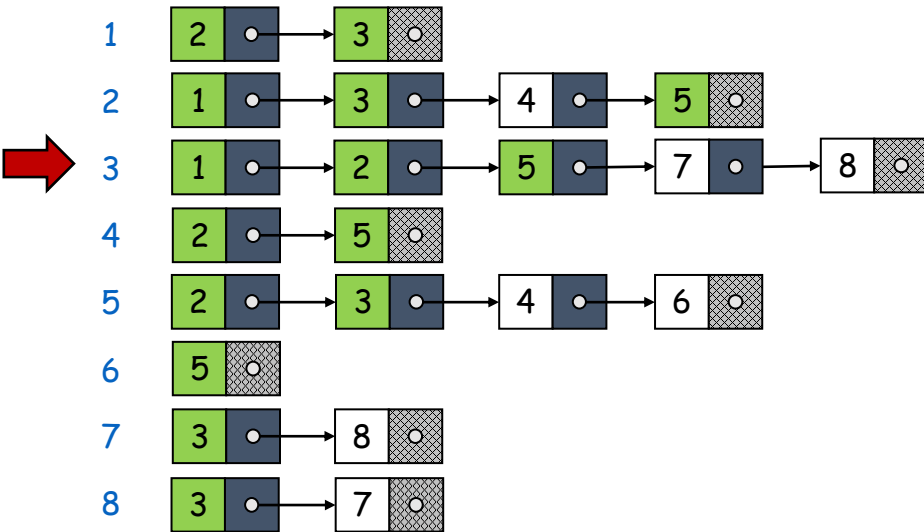
1

2

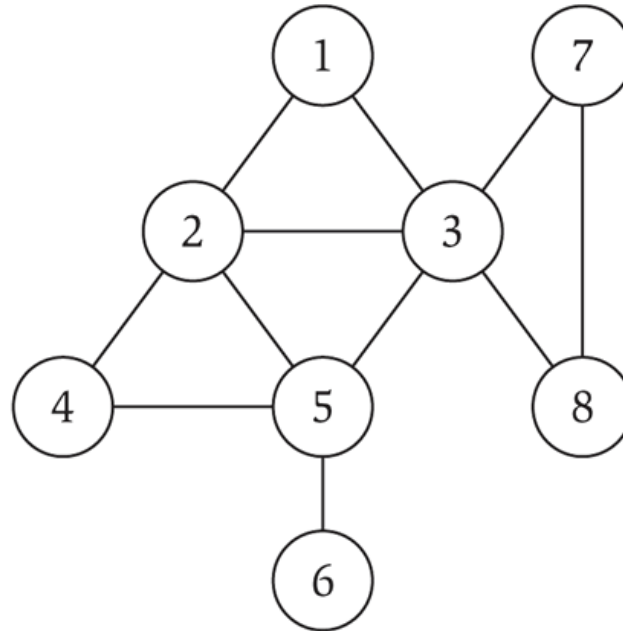
3

5

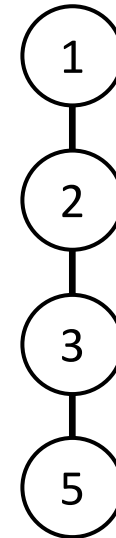
Adjacency List



Graph



DFS Tree



Stack

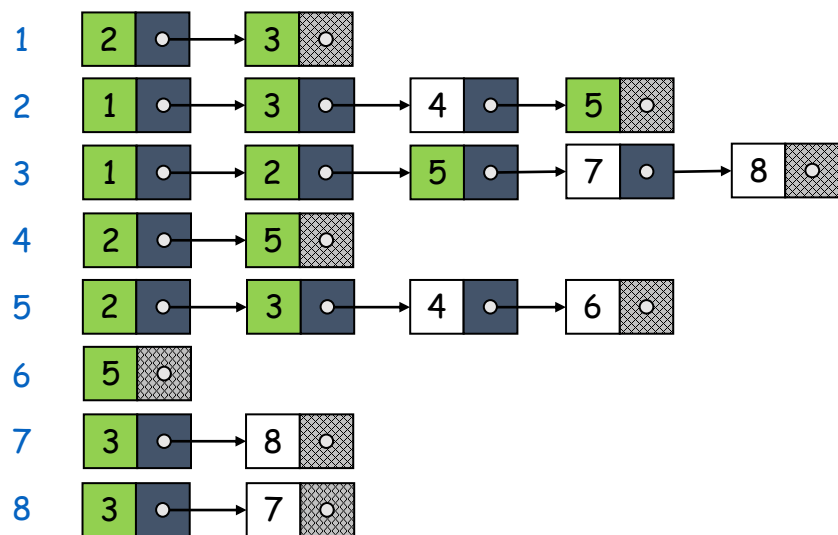
1

2

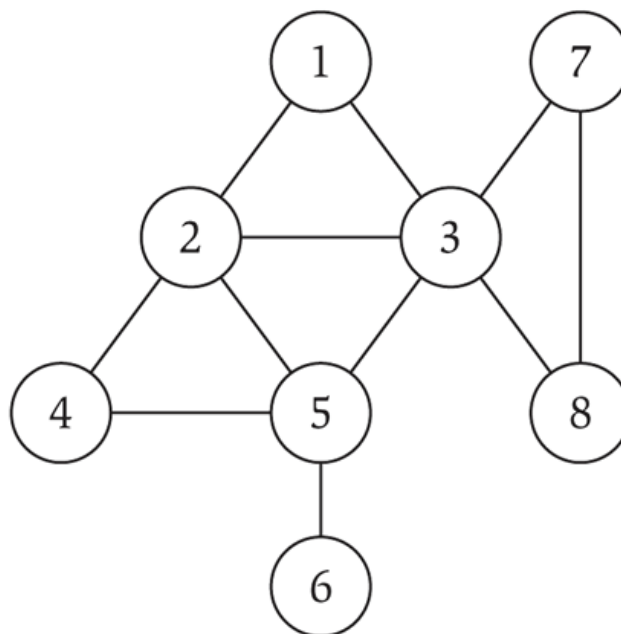
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5

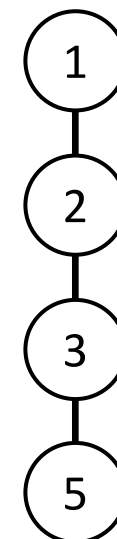
Adjacency List



Graph



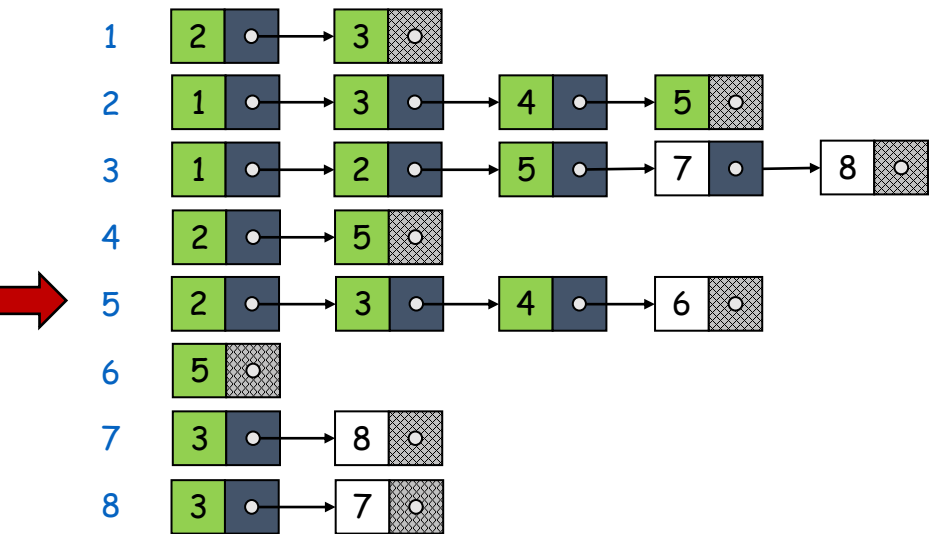
DFS Tree



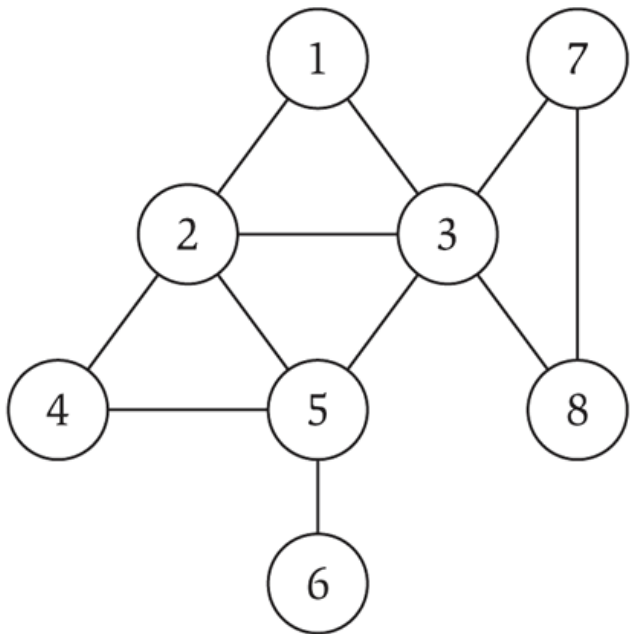
Stack



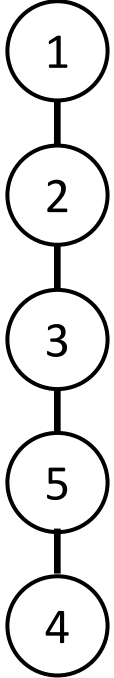
Adjacency List



Graph



DFS Tree



Stack

1

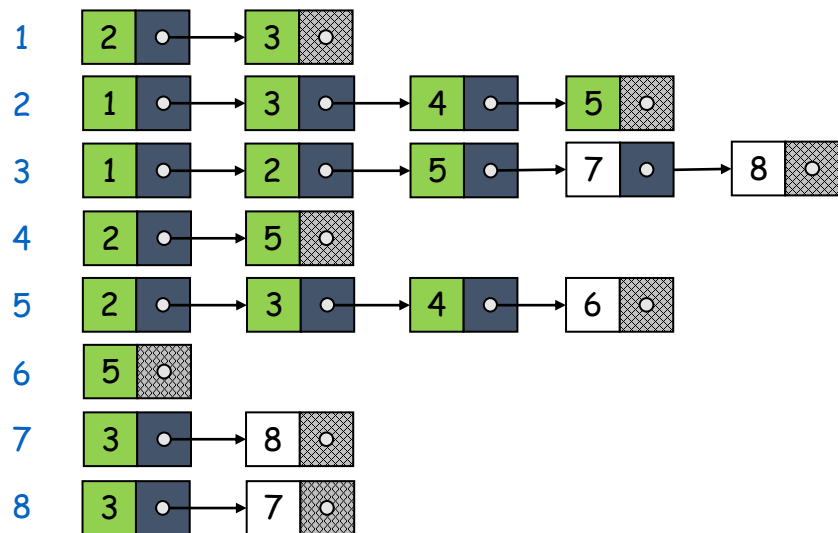
2

3

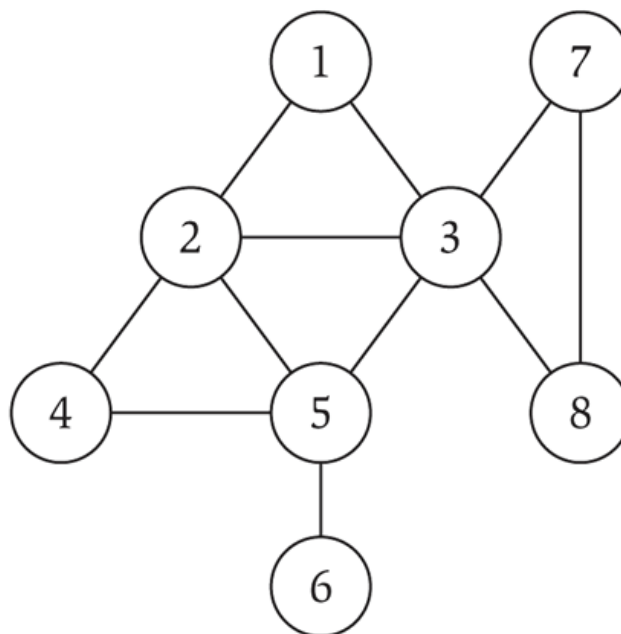
5

4

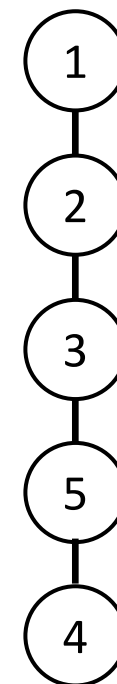
Adjacency List



Graph



DFS Tree



Stack

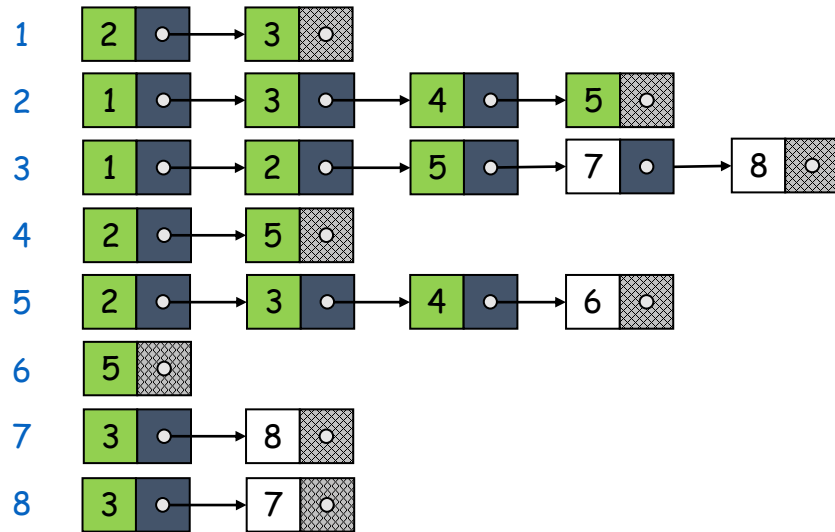
1

2

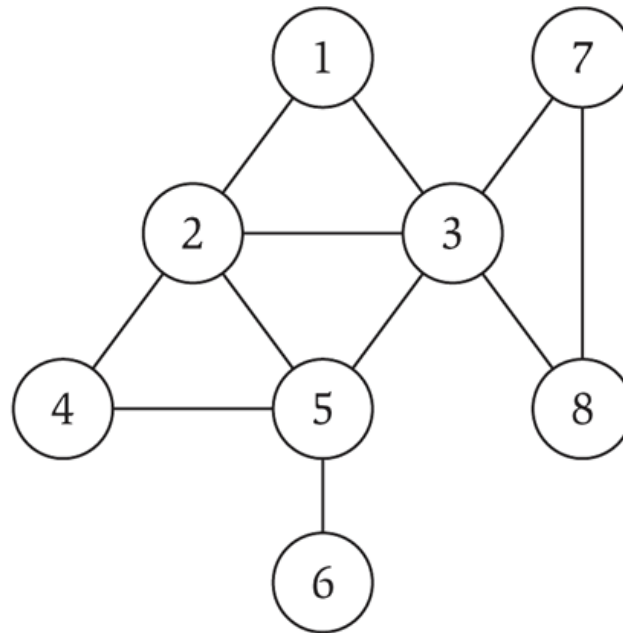
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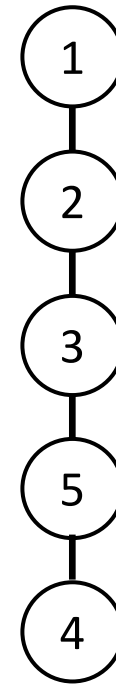
Adjacency List



Graph



DFS Tree



Stack

1

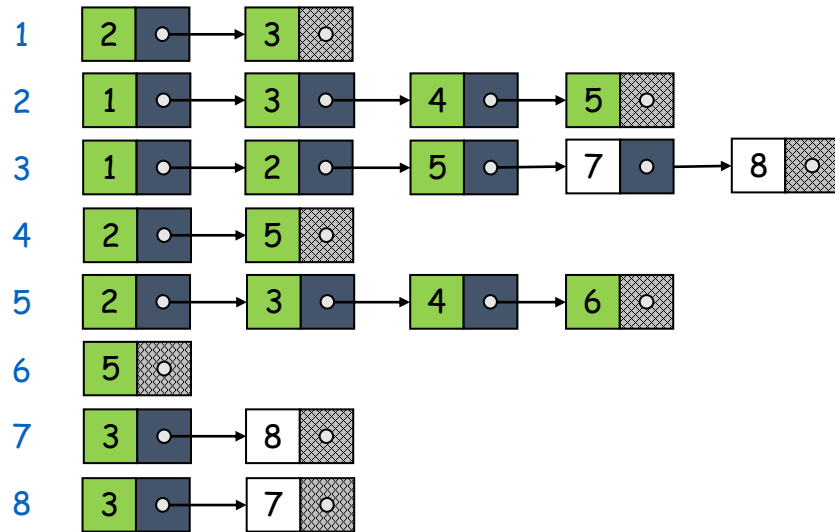
2

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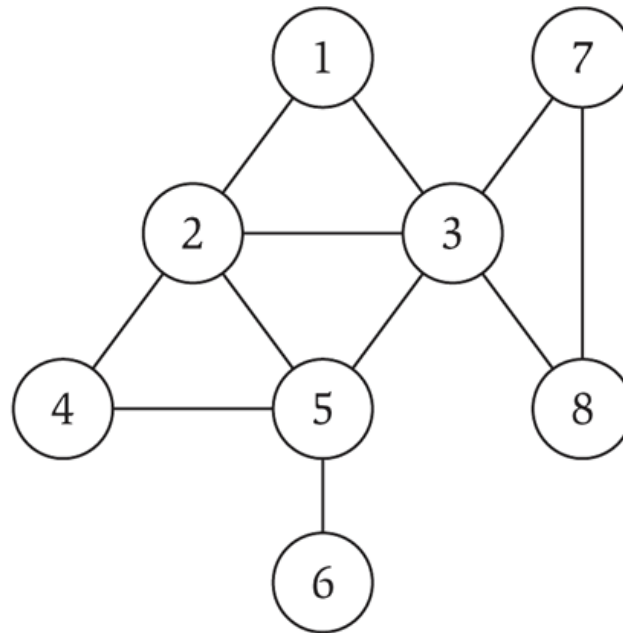
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6

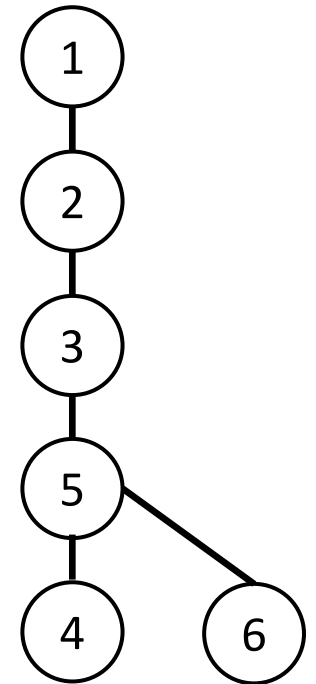
Adjacency List



Graph



DFS Tree



Stack

1

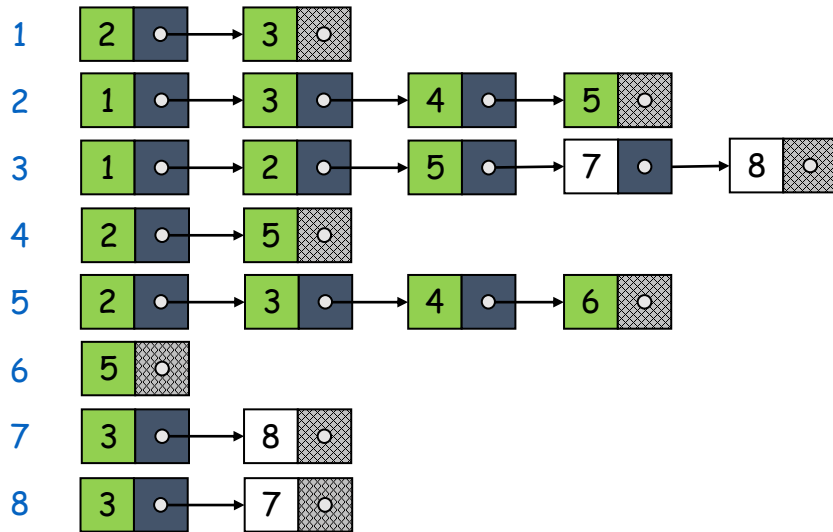
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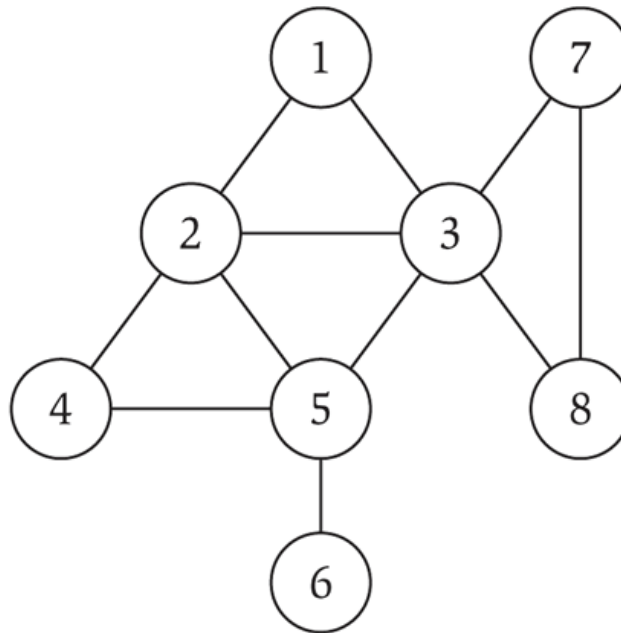
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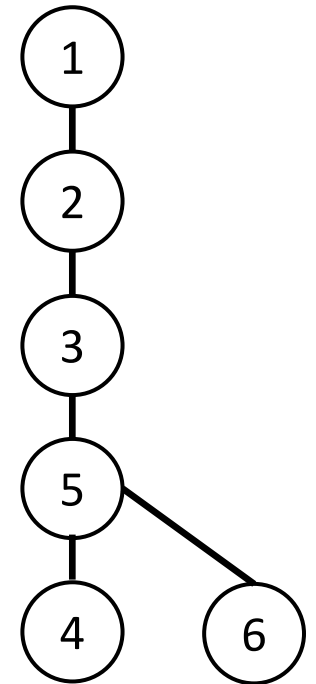
Adjacency List



Graph



DFS Tree



Stack

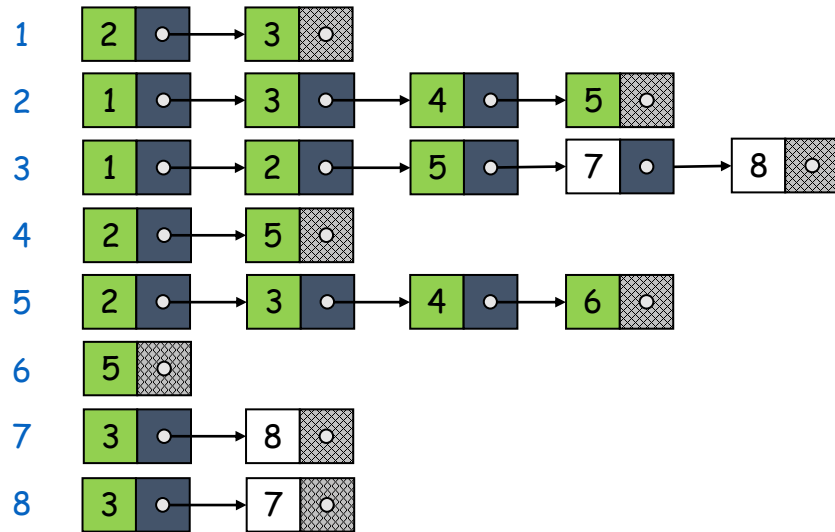
1

2

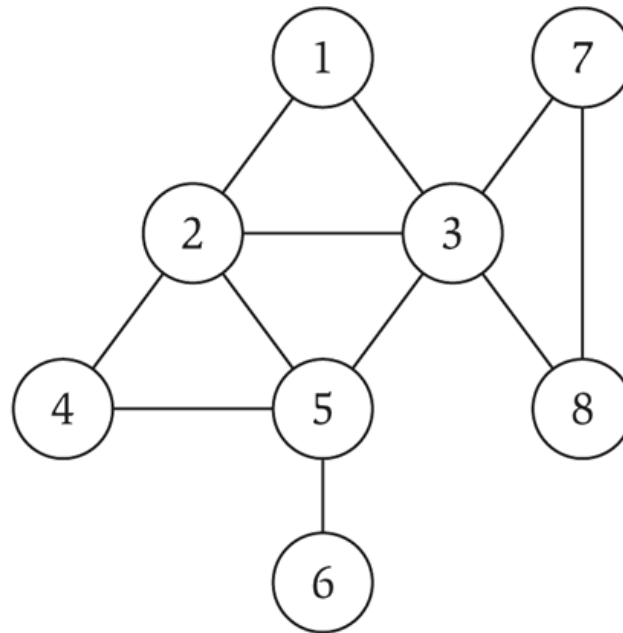
3

5

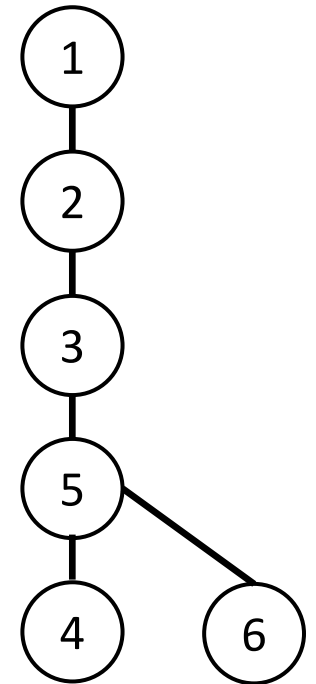
Adjacency List



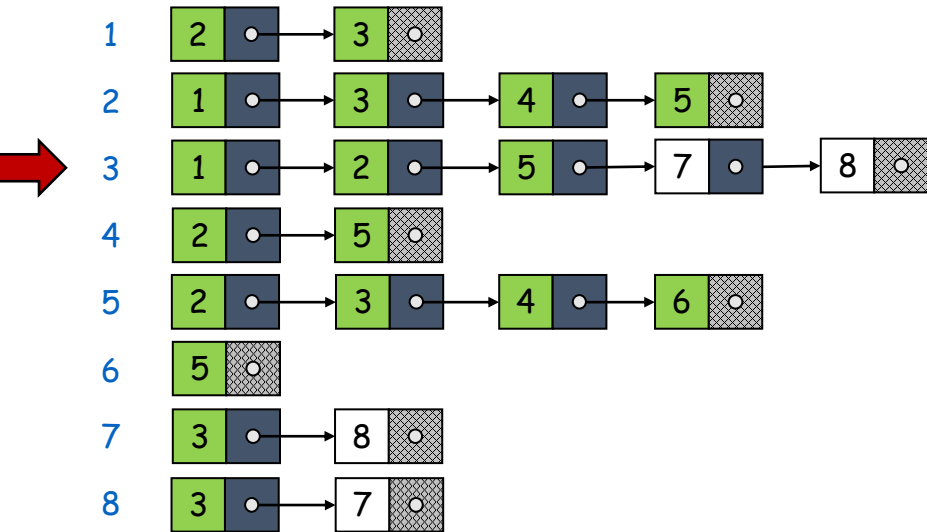
Graph



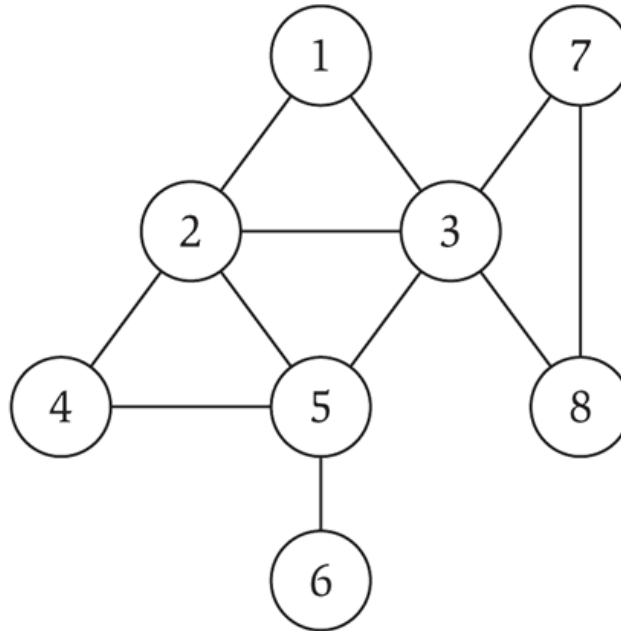
DFS Tree



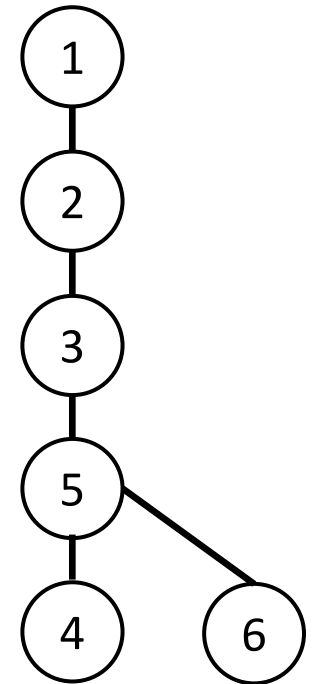
Adjacency List



Graph



DFS Tree



Stack

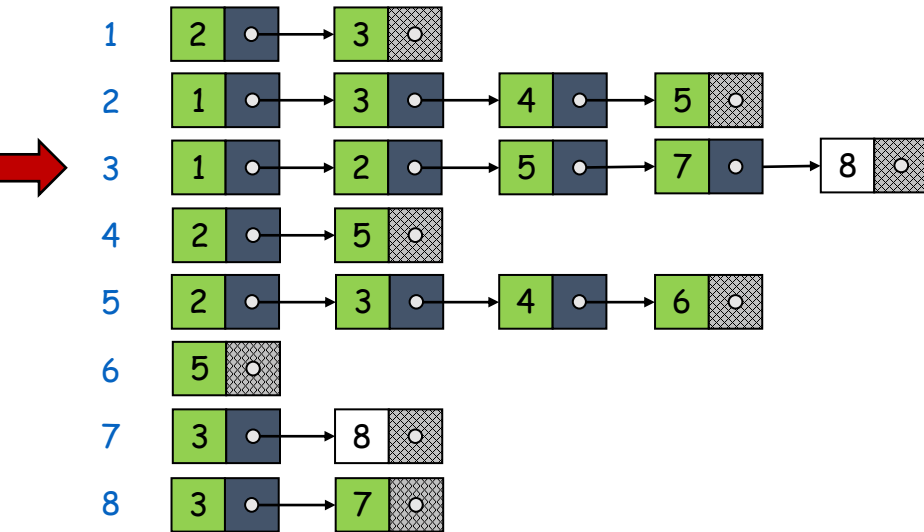
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2

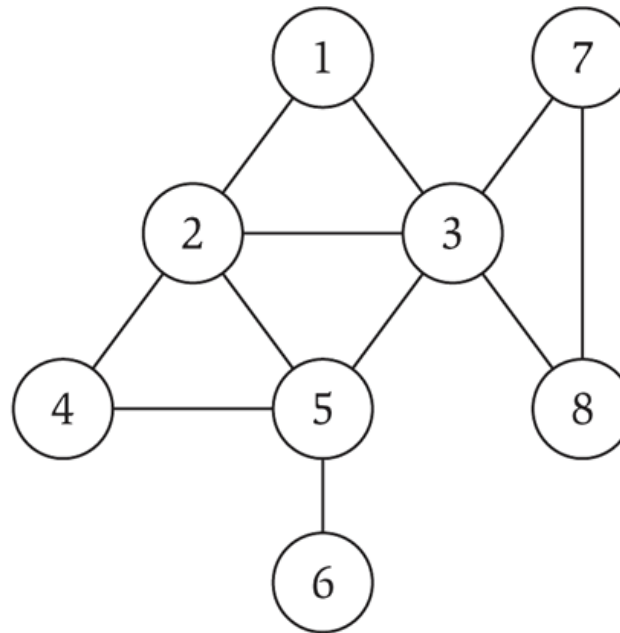
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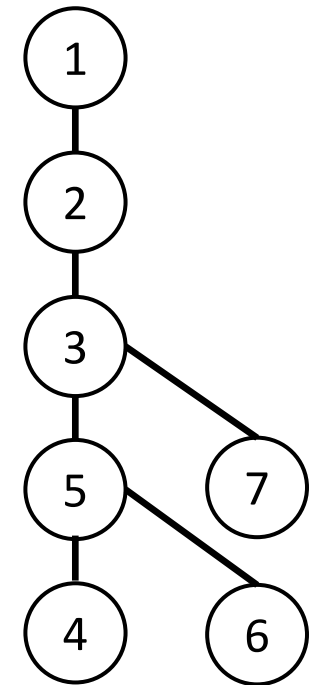
Adjacency List



Graph



DFS Tree



Stack

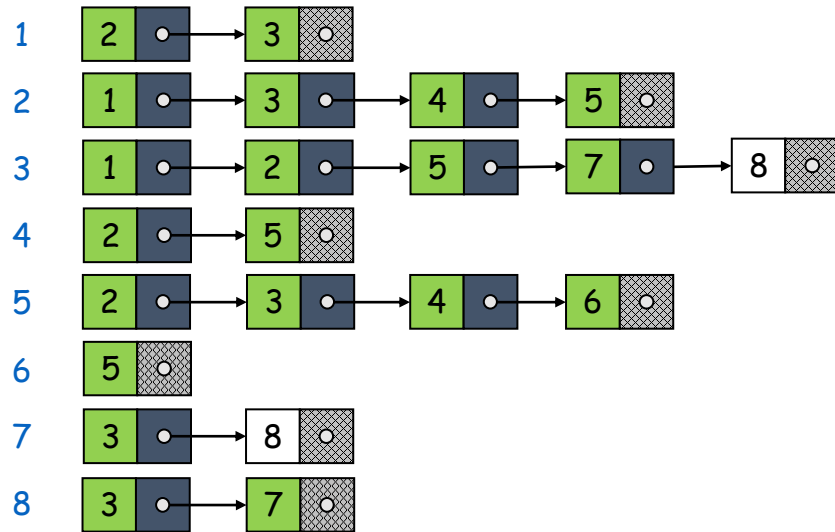
1

2

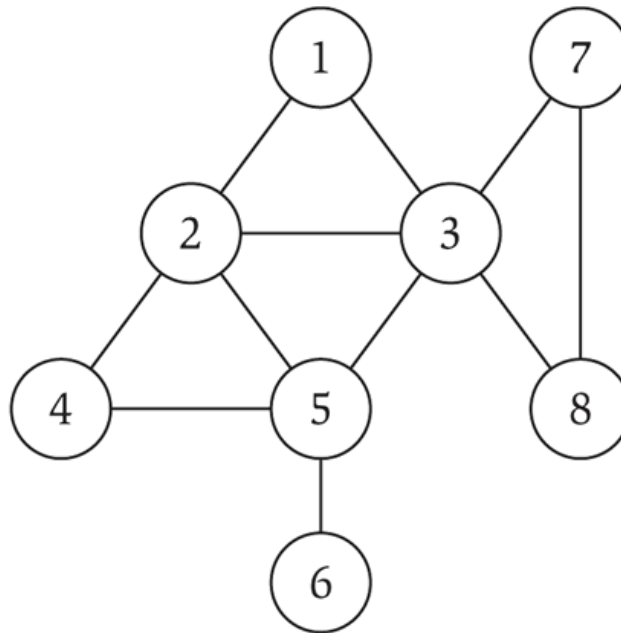
3

7

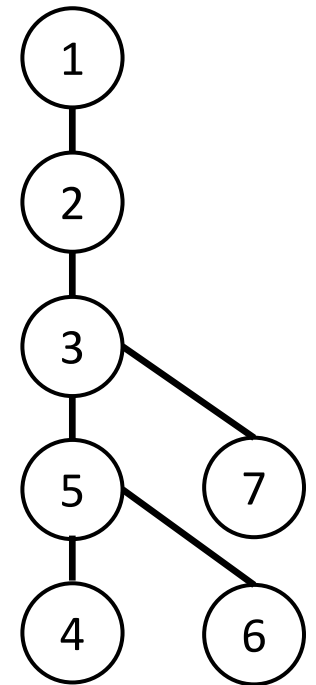
Adjacency List



Graph



DFS Tree



Stack

1

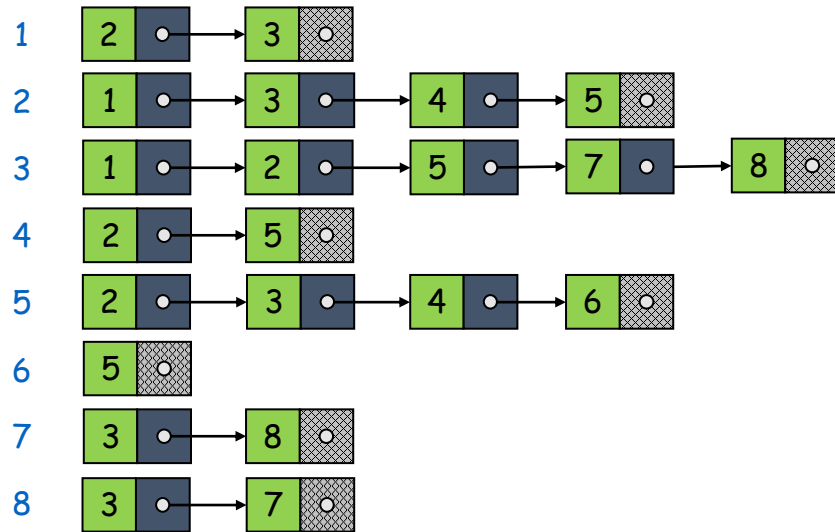
2

3

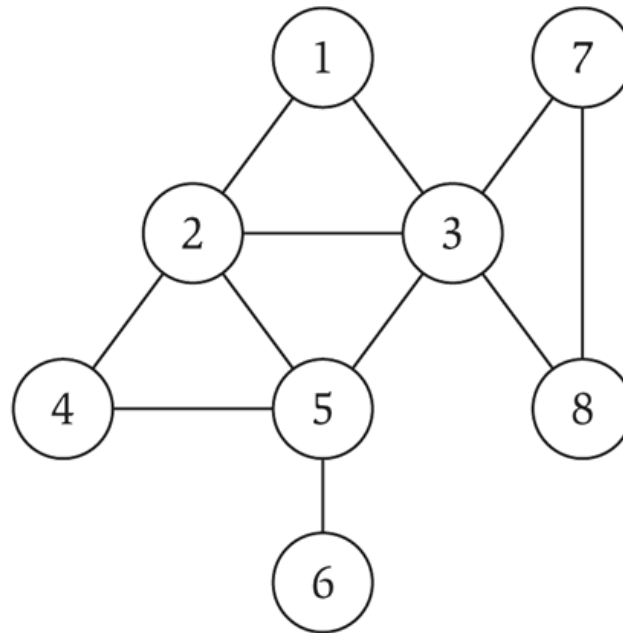
7

8

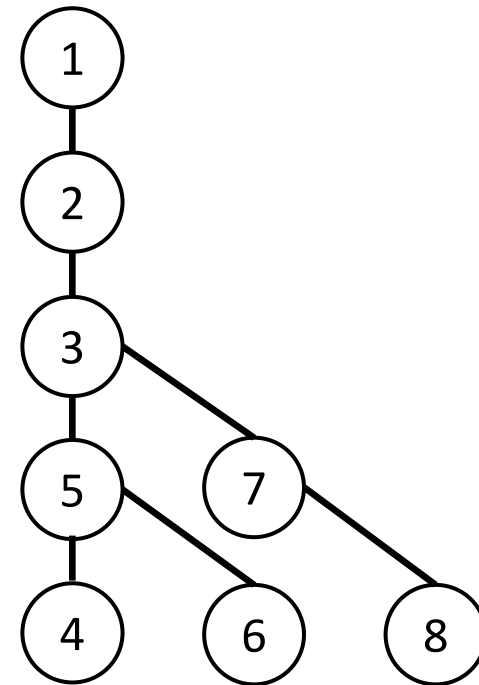
Adjacency List



Graph



DFS Tree



Stack

1

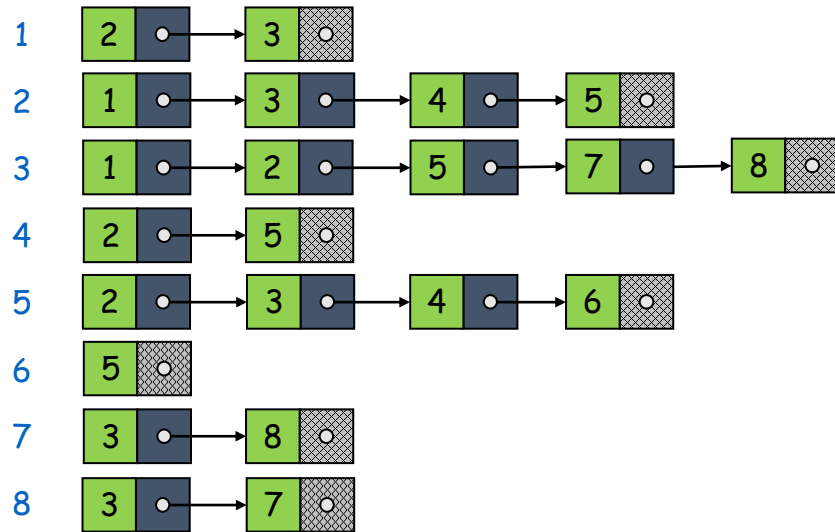
2

3

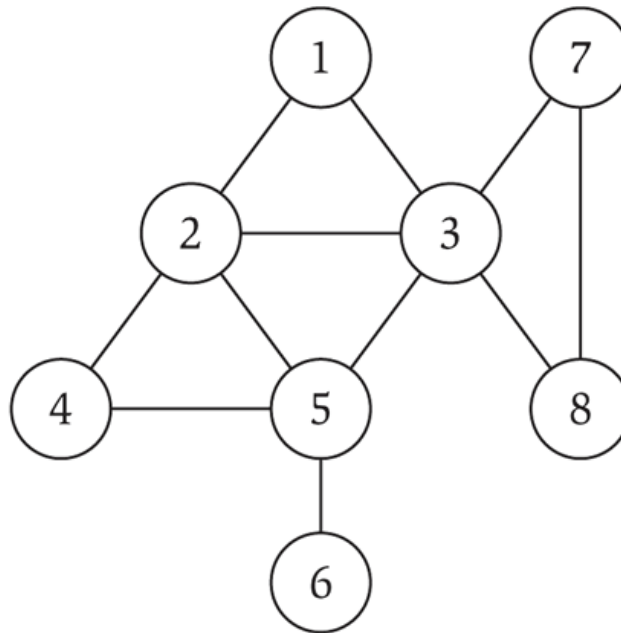
7

8

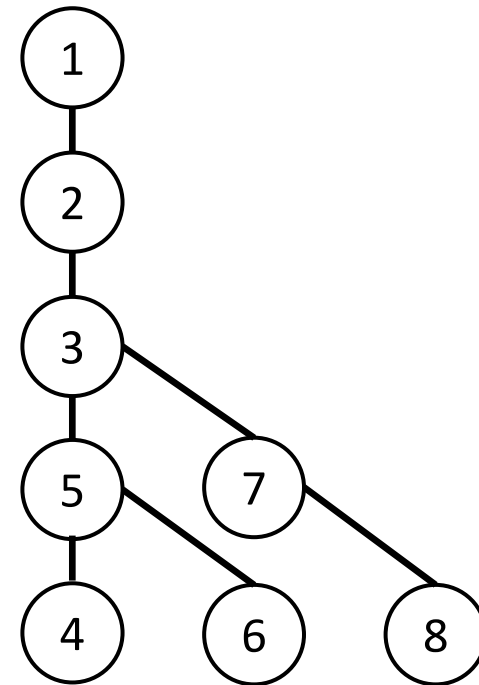
Adjacency List



Graph



DFS Tree



Stack

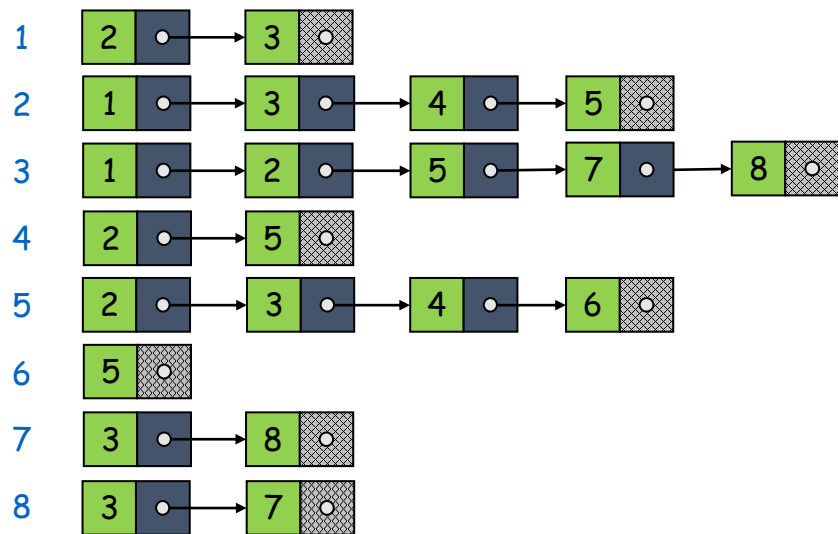
1

2

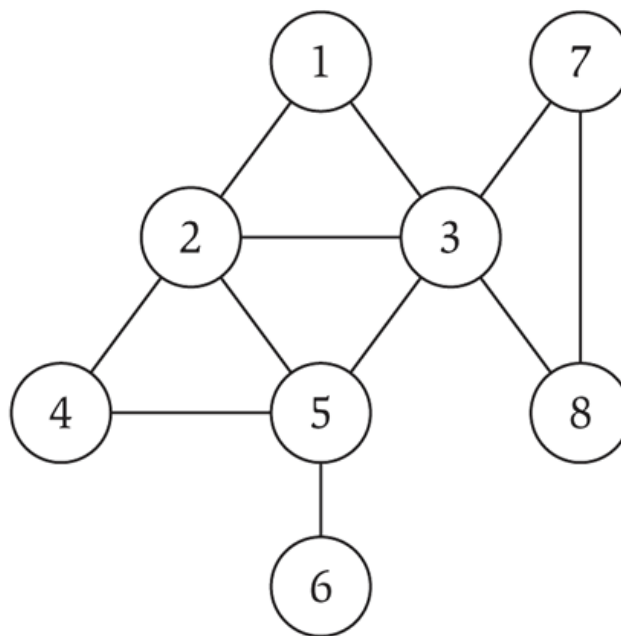
3

7

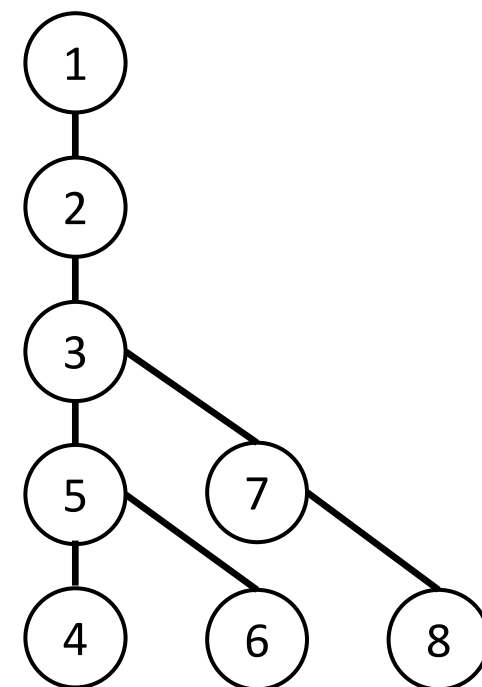
Adjacency List



Graph



DFS Tree



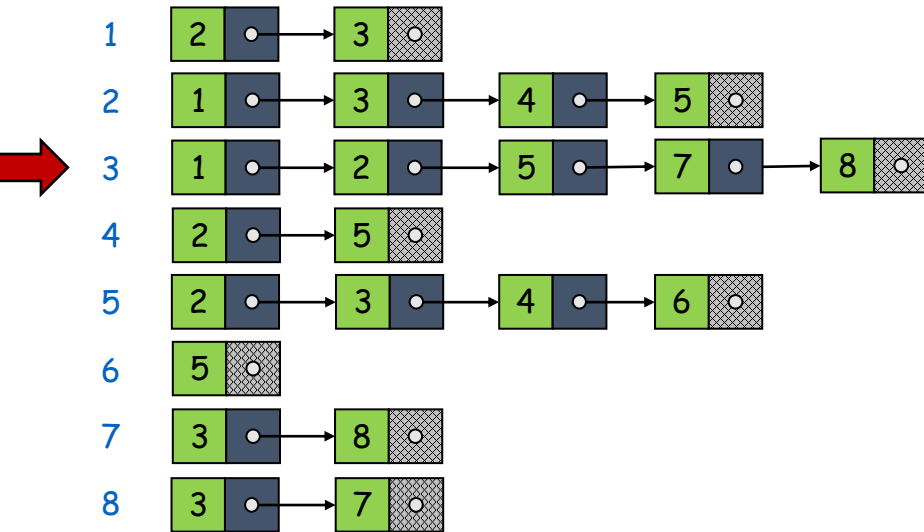
Stack

1

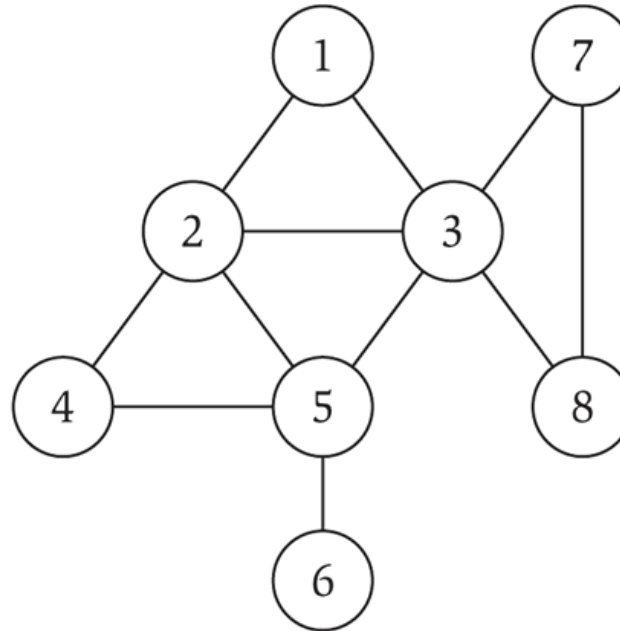
2

3

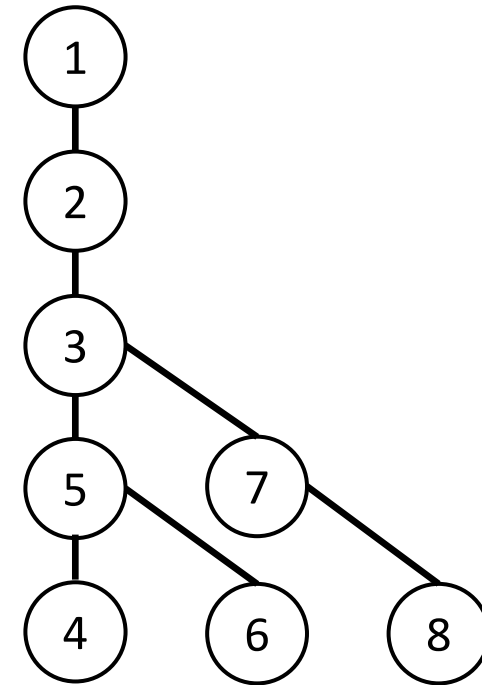
Adjacency List



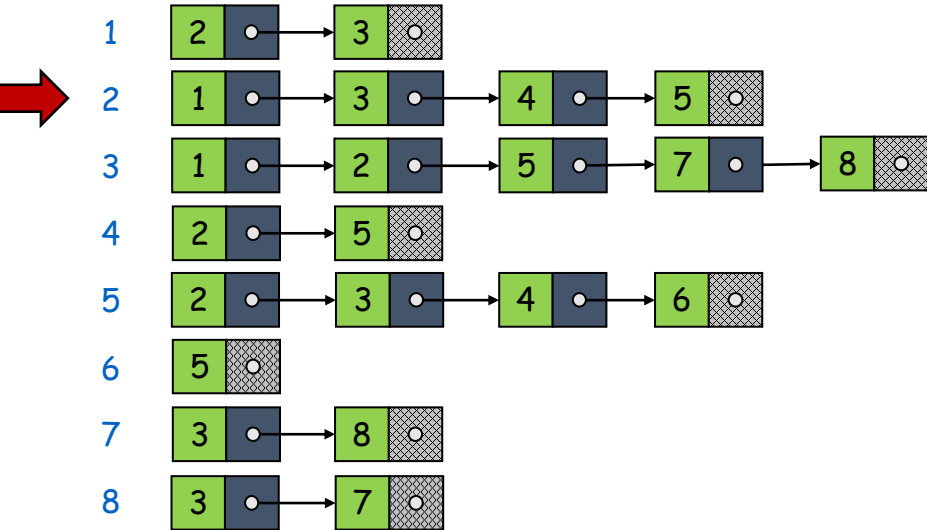
Graph



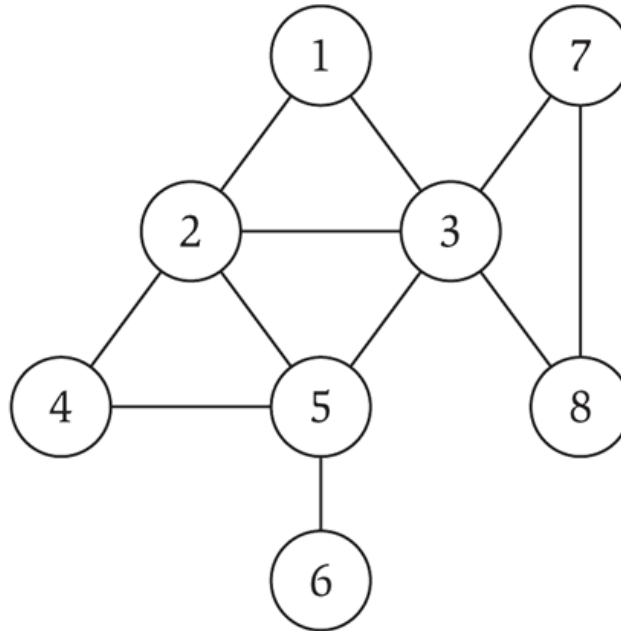
DFS Tree



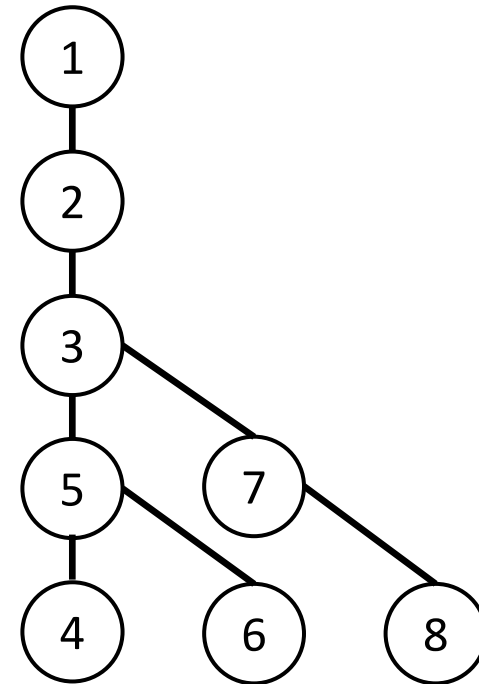
Adjacency List



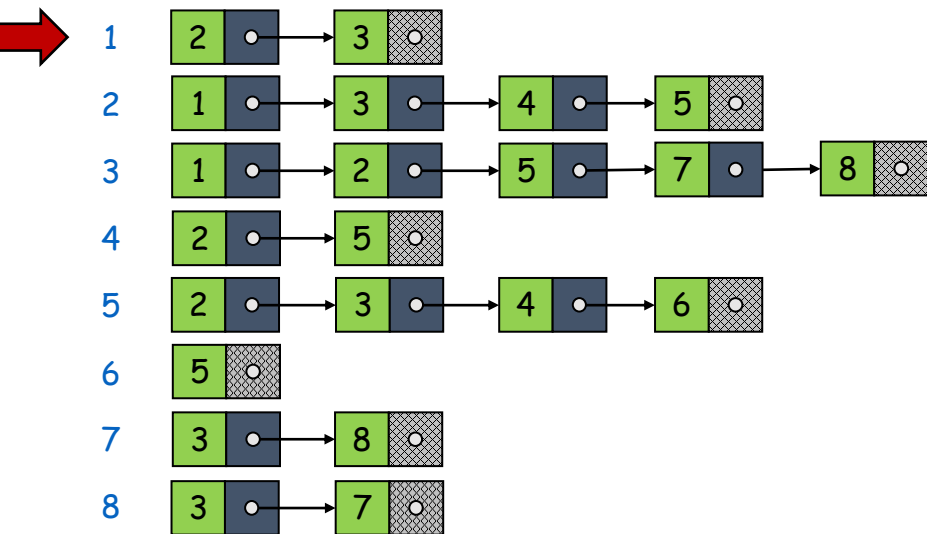
Graph



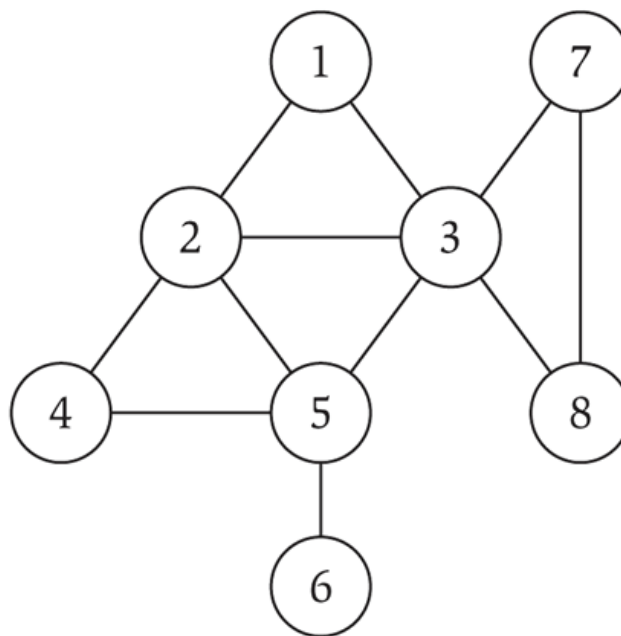
DFS Tree



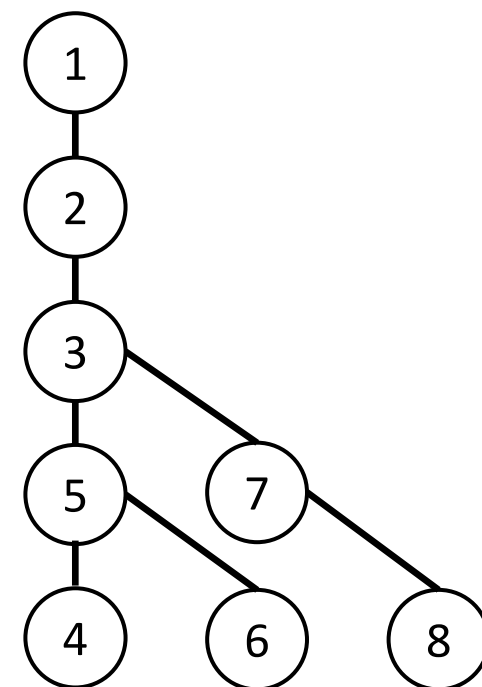
Adjacency List



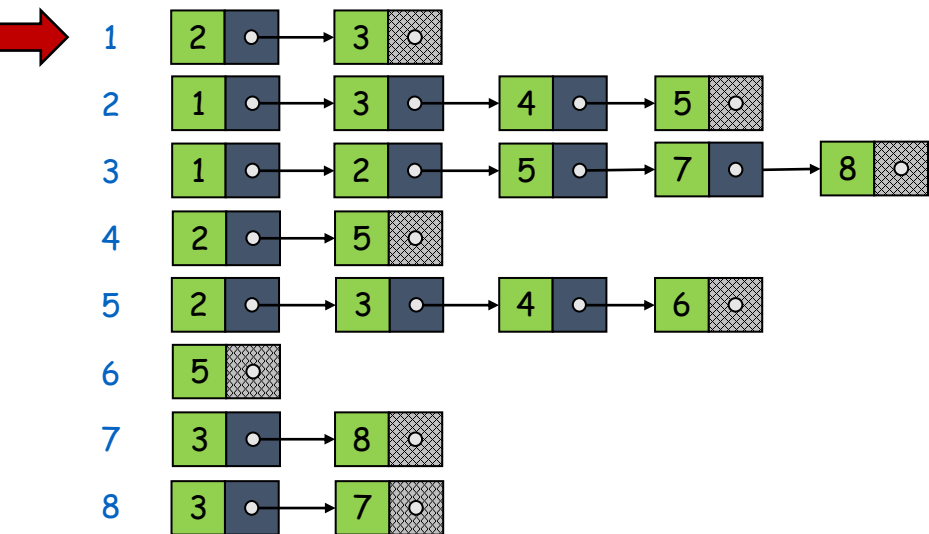
Graph



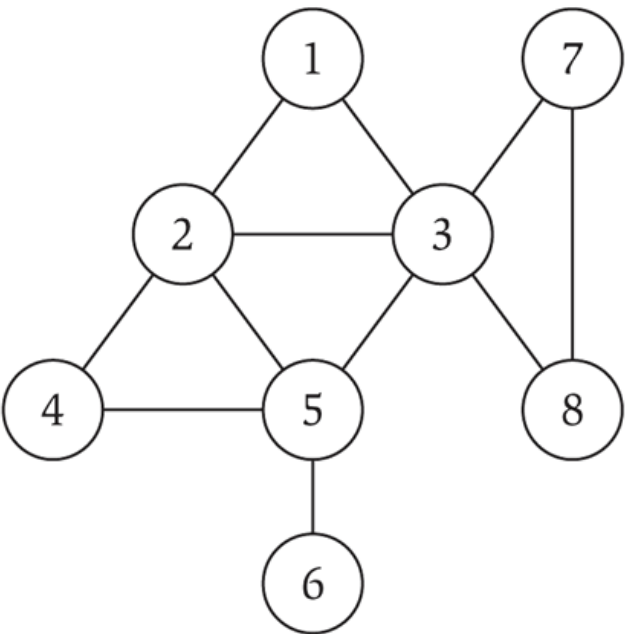
DFS Tree



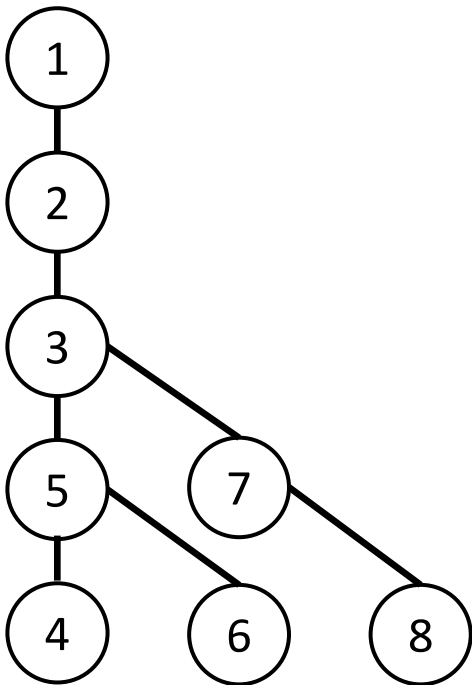
Adjacency List



Graph



DFS Tree



Depth First Search (DFS) – *via Recursion*

- Given a directed Graph, $G = (V, E)$, DFS traverses all vertices of G
 - Each vertex v is labelled with a discovery time $d.v$ and finish time $f.v$
 - $d.v$ denotes the time when vertex v is first processed
 - $f.v$ denotes the time when all of the descendants of v are finished, i.e., when all the paths originating from v are processed.
- DFS colors the vertices during the search to indicate their state
 - Initially all vertices are colored **white**
 - *White* vertices are not discovered yet
 - When a vertex is discovered, it is colored **gray**
 - *Gray* vertices are discovered, however all paths originating from a gray vertex are not explored yet
 - When a vertex is finished it is colored **black**
 - All paths from a *black* vertex are explored

Extra
Bookkeeping

$$G = (V, E)$$

DFS(G)

```
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7          DFS-VISIT( $G, u$ )
```

DFS-VISIT(G, u)

```
1   $time = time + 1$ 
2   $u.d = time$ 
3   $u.color = GRAY$ 
4  for each  $v \in Adj[u]$ 
5      if  $v.color == WHITE$ 
6           $v.\pi = u$ 
7          DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
```

$$G = (V, E)$$

DFS(G)

```

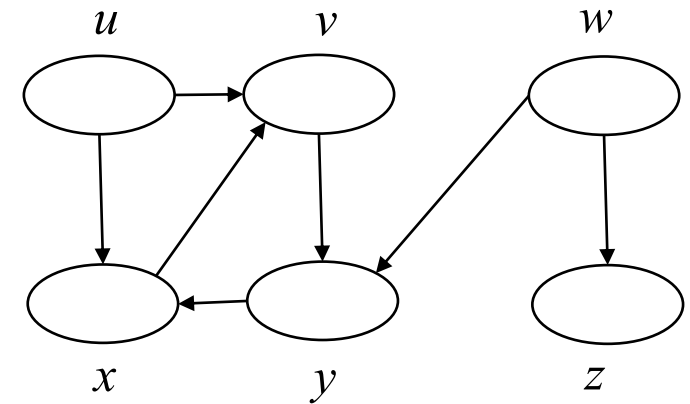
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```

Example



$$G = (V, E)$$

DFS(G)

```

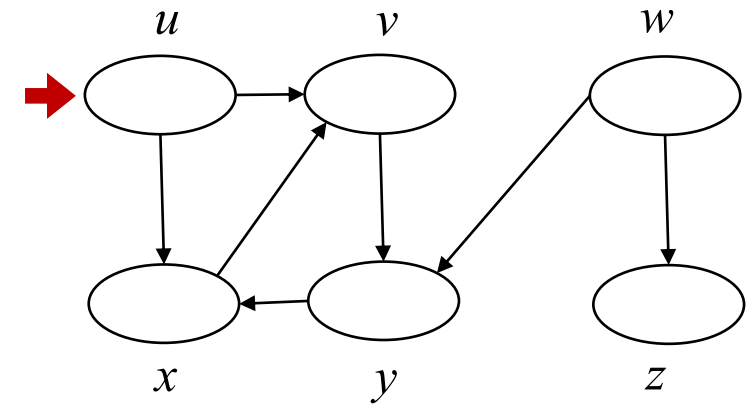
1  for each vertex  $u \in V$ 
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4   $time = 0$ 
5  → for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
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DFS-VISIT(G, u)

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```

Example



$$G = (V, E)$$

DFS(G)

```

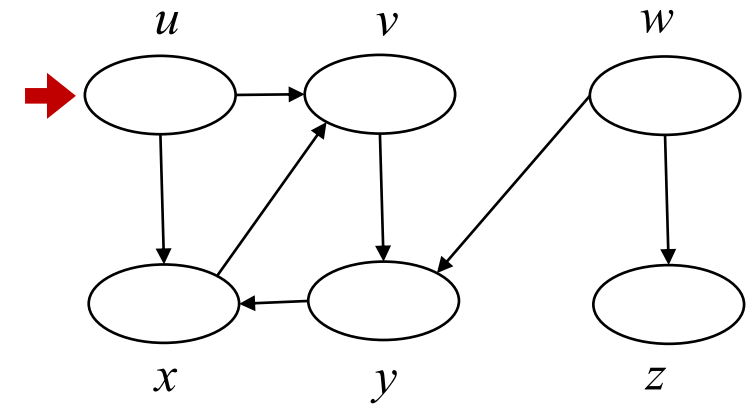
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Example



$$G = (V, E)$$

DFS(G)

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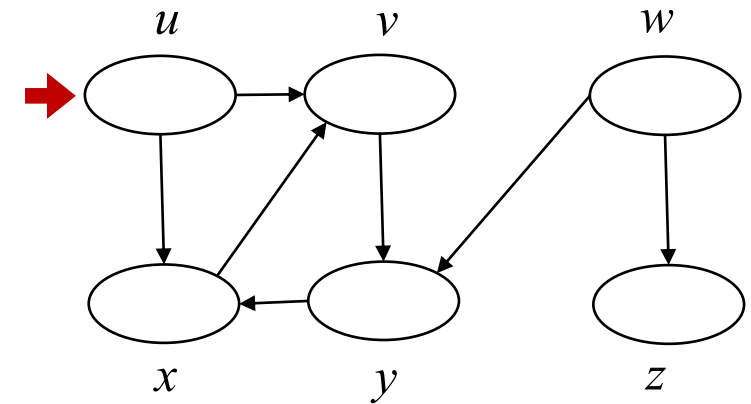
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```

Example



$$G = (V, E)$$

DFS(G)

```

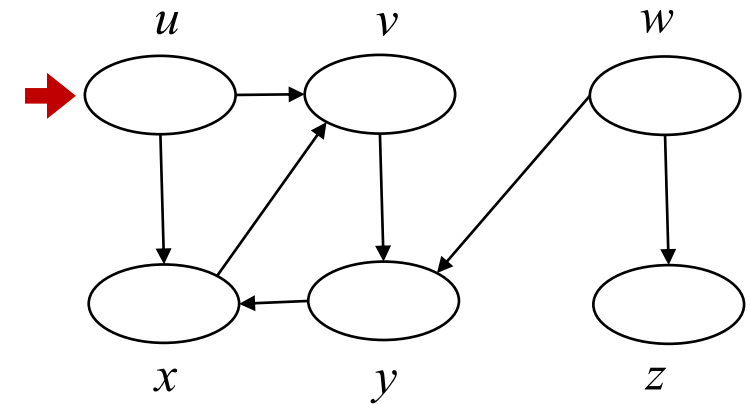
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```

Example



$$G = (V, E)$$

DFS(G)

```

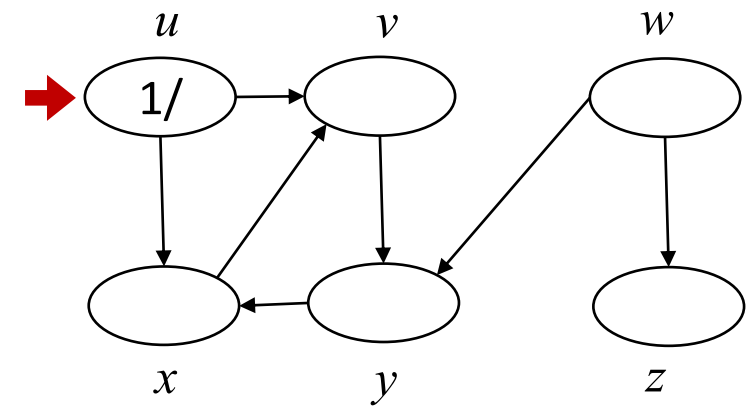
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Example



$$G = (V, E)$$

DFS(G)

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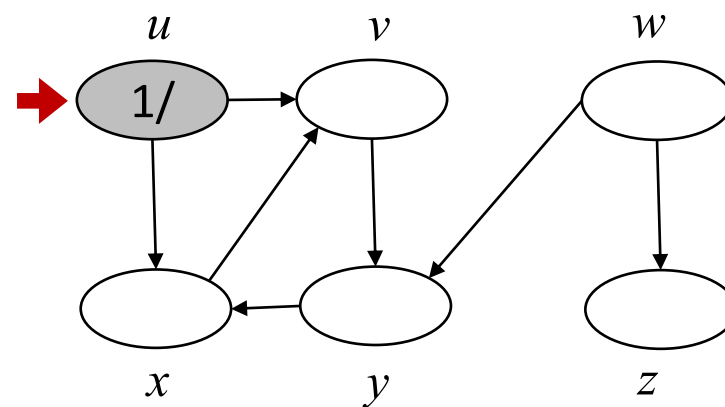
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```

Example



$$G = (V, E)$$

DFS(G)

```

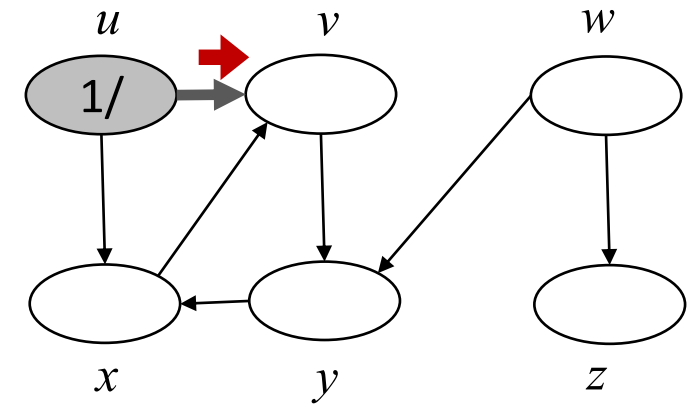
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DFS-VISIT(G, u)

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Example



$$G = (V, E)$$

DFS(G)

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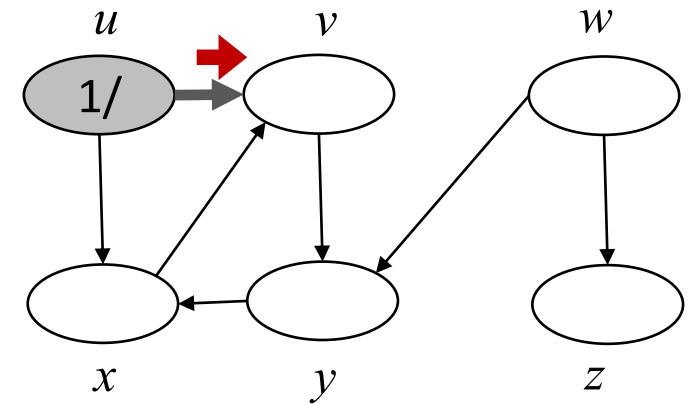
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```

Example



$$G = (V, E)$$

DFS(G)

```

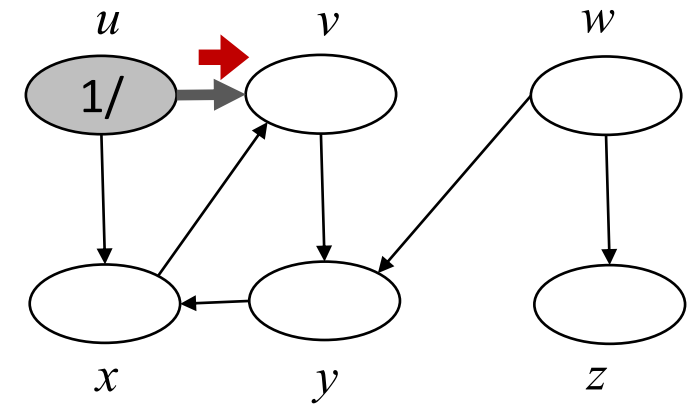
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Example



$$G = (V, E)$$

DFS(G)

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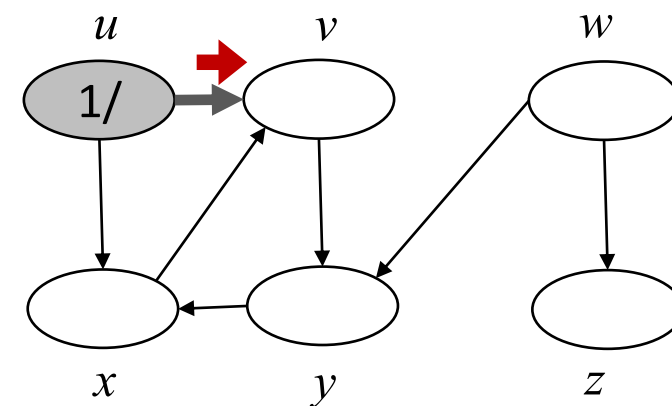
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Example



$$G = (V, E)$$

DFS(G)

```

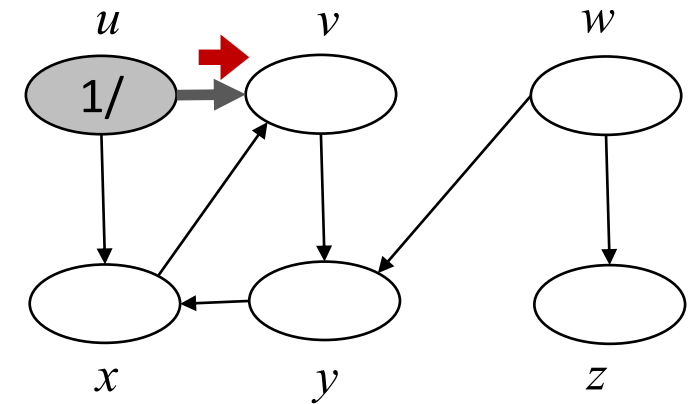
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```

Example



$$G = (V, E)$$

DFS(G)

```

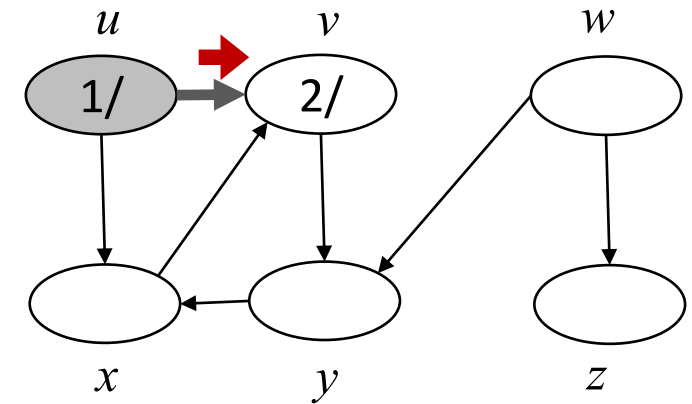
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Example



$$G = (V, E)$$

DFS(G)

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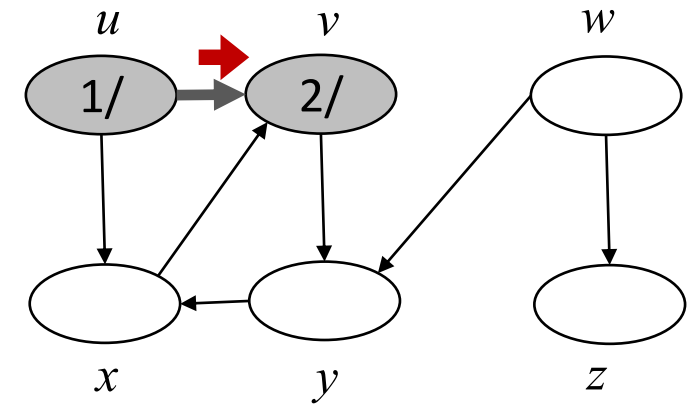
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Example



$$G = (V, E)$$

DFS(G)

```

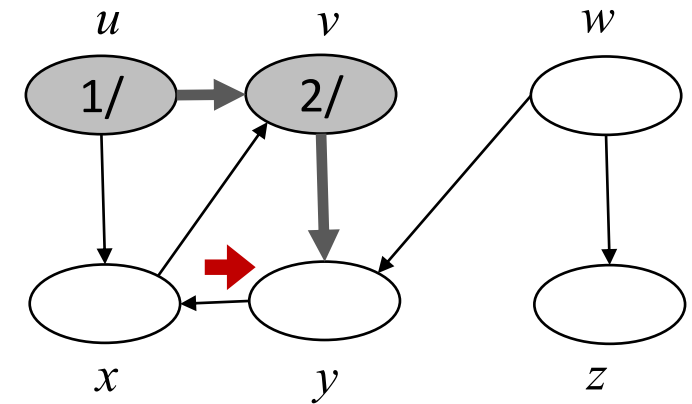
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Example



$$G = (V, E)$$

DFS(G)

```

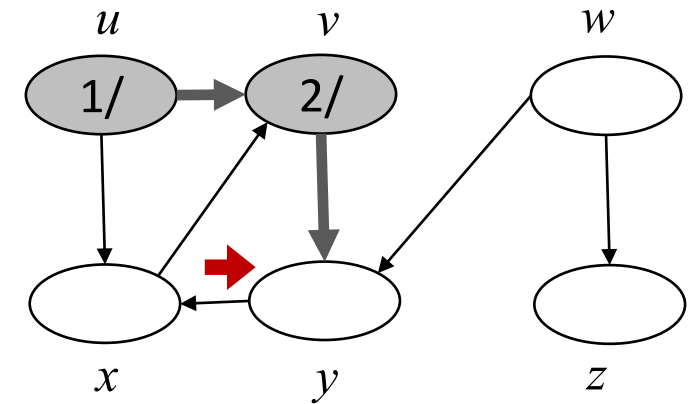
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```

Example



$$G = (V, E)$$

DFS(G)

```

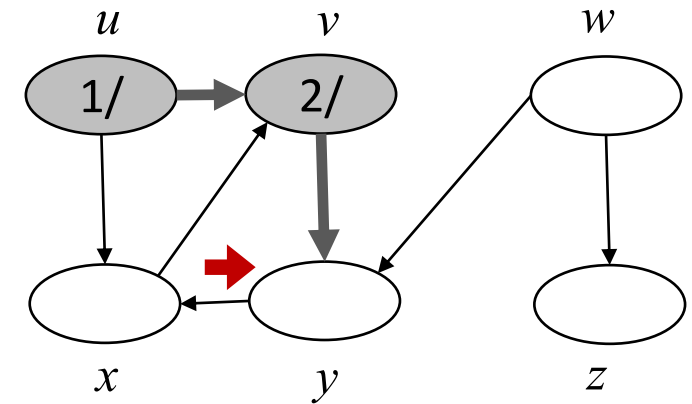
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7   $\rightarrow$       DFS-VISIT( $G, u$ )
  
```

DFS-VISIT(G, u)

```

1   $time = time + 1$ 
2   $u.d = time$ 
3   $u.color = GRAY$ 
4  for each  $v \in Adj[u]$ 
5      if  $v.color == WHITE$ 
6   $\rightarrow$        $v.\pi = u$ 
7          DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
  
```

Example



$$G = (V, E)$$

DFS(G)

```

1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
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7   $\rightarrow$       DFS-VISIT( $G, u$ )

```

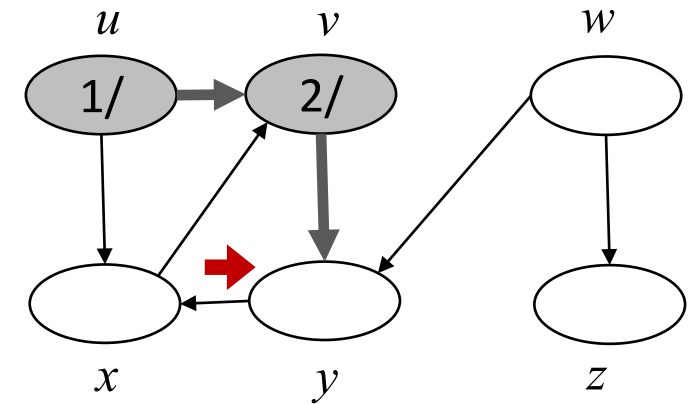
DFS-VISIT(G, u)

```

1   $time = time + 1$ 
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3   $u.color = GRAY$ 
4  for each  $v \in Adj[u]$ 
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8   $u.color = BLACK$ 
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10  $u.f = time$ 

```

Example



$$G = (V, E)$$

DFS(G)

```

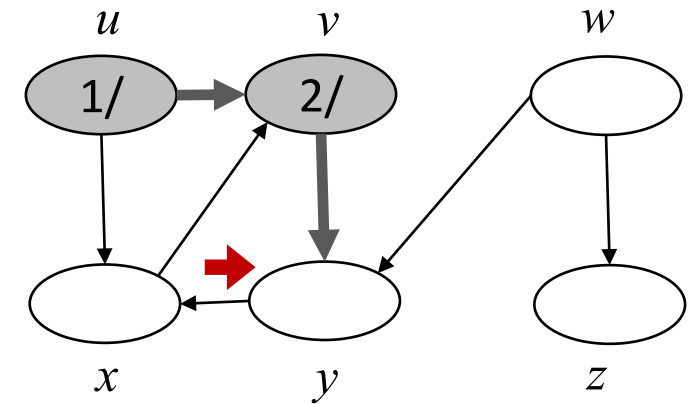
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
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4   $time = 0$ 
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8   $u.color = BLACK$ 
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10  $u.f = time$ 
    
```

Example



$$G = (V, E)$$

DFS(G)

```

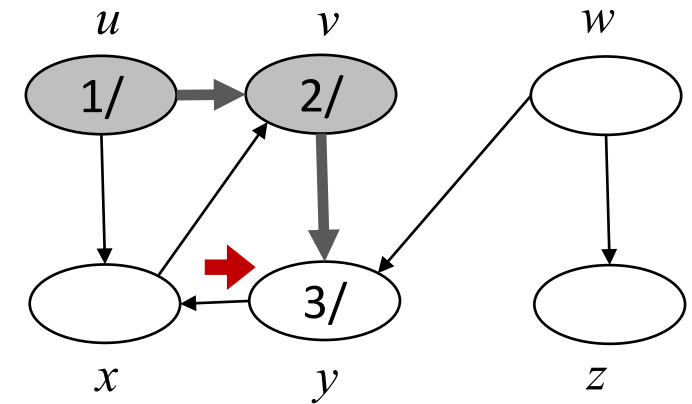
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
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DFS-VISIT(G, u)

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8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
    
```

Example



$$G = (V, E)$$

DFS(G)

```

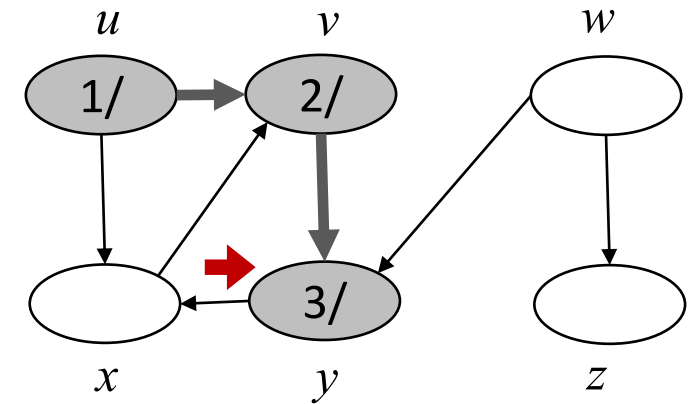
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
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6      if  $u.color == WHITE$ 
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DFS-VISIT(G, u)

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1   $time = time + 1$ 
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8   $u.color = BLACK$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

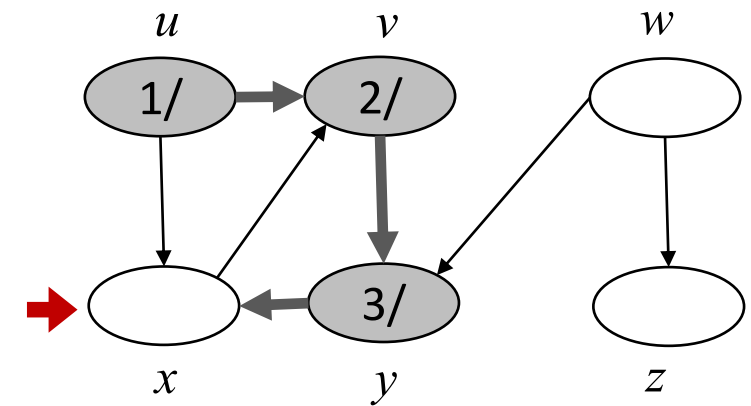
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
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DFS-VISIT(G, u)

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8   $u.color = BLACK$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

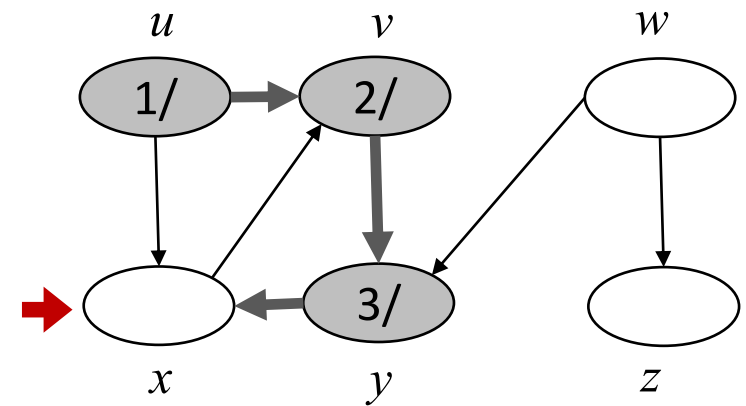
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7   $\rightarrow$       DFS-VISIT( $G, u$ )
    
```

DFS-VISIT(G, u)

```

1   $time = time + 1$ 
2   $u.d = time$ 
3   $u.color = GRAY$ 
4  for each  $v \in Adj[u]$ 
5   $\rightarrow$       if  $v.color == WHITE$ 
6               $v.\pi = u$ 
7              DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
    
```

Example



$$G = (V, E)$$

DFS(G)

```

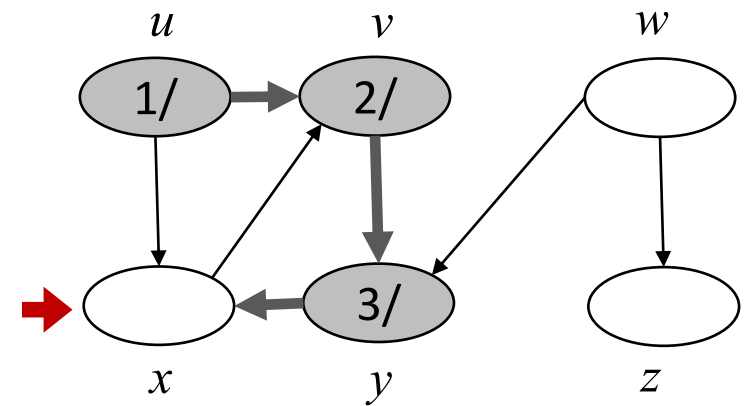
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7   $\rightarrow$       DFS-VISIT( $G, u$ )
  
```

DFS-VISIT(G, u)

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1   $time = time + 1$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

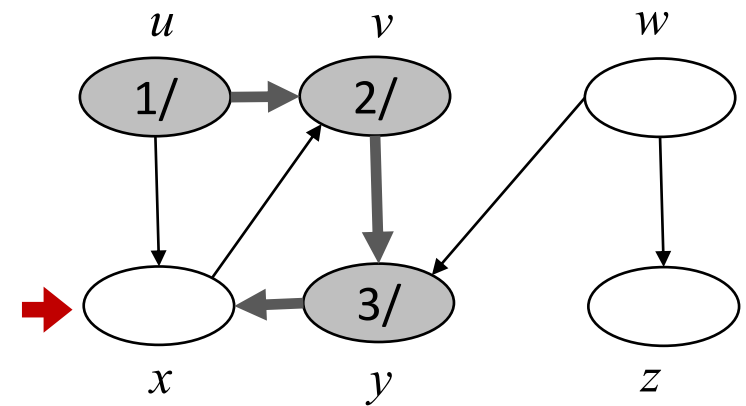
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DFS-VISIT(G, u)

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8   $u.color = BLACK$ 
9   $time = time + 1$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

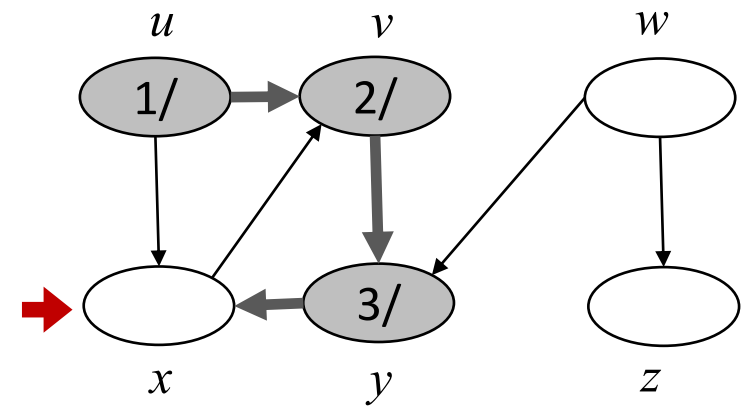
1  for each vertex  $u \in V$ 
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```

DFS-VISIT(G, u)

```

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6           $v.\pi = u$ 
7          DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
    
```

Example



$$G = (V, E)$$

DFS(G)

```

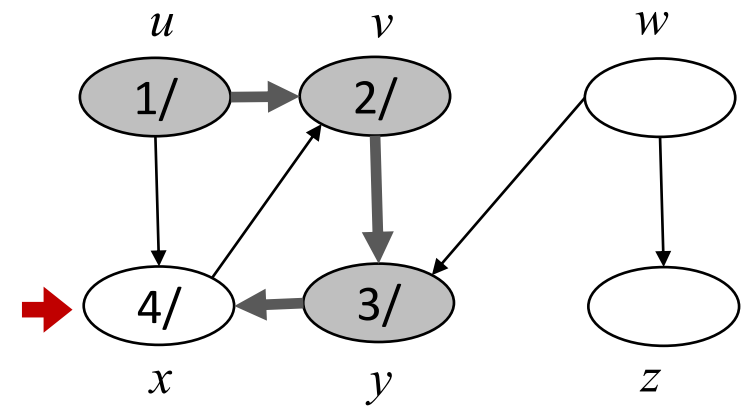
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7   $\rightarrow$       DFS-VISIT( $G, u$ )
    
```

DFS-VISIT(G, u)

```

1   $time = time + 1$ 
2   $\rightarrow$   $u.d = time$ 
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4  for each  $v \in Adj[u]$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

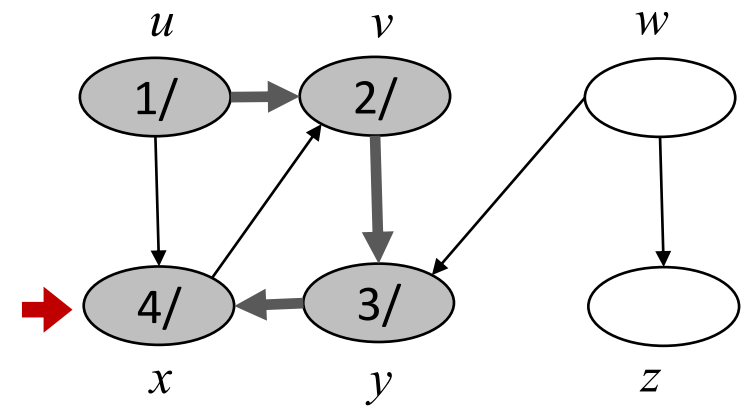
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
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DFS-VISIT(G, u)

```

1   $time = time + 1$ 
2   $u.d = time$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

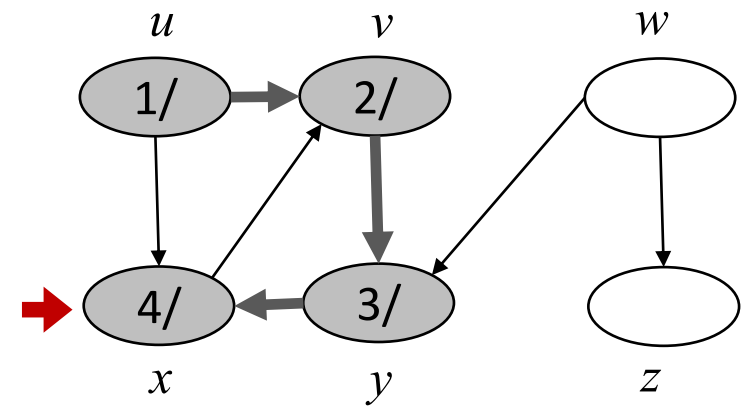
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
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6      if  $u.color == WHITE$ 
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DFS-VISIT(G, u)

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8   $u.color = BLACK$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

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4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7  →      DFS-VISIT( $G, u$ )

```

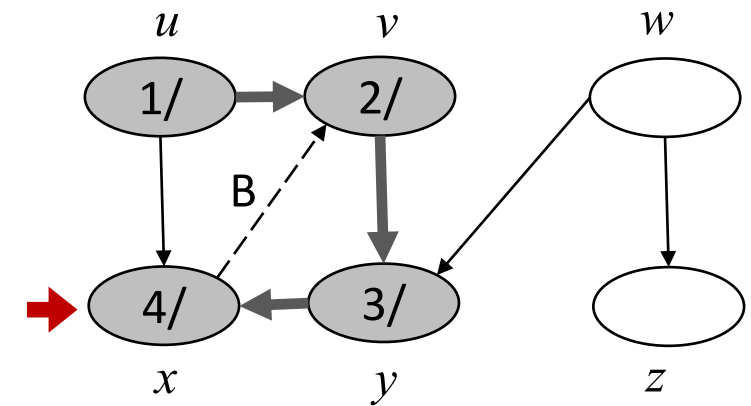
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1   $time = time + 1$ 
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7              DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 

```

Example



Back edges are those edges (u, v) connecting a vertex u to an ancestor vertex v in a depth-first tree. Self-loop (edge (u, u)), is also considered as Back edge

$$G = (V, E)$$

DFS(G)

```

1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
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6      if  $u.color == WHITE$ 
7   $\rightarrow$       DFS-VISIT( $G, u$ )

```

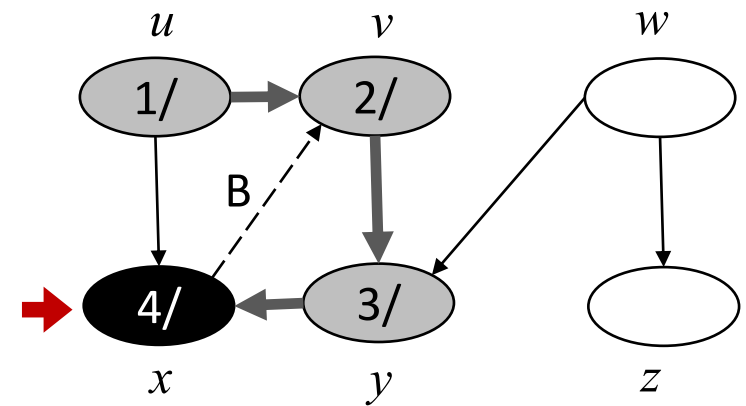
DFS-VISIT(G, u)

```

1   $time = time + 1$ 
2   $u.d = time$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

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```

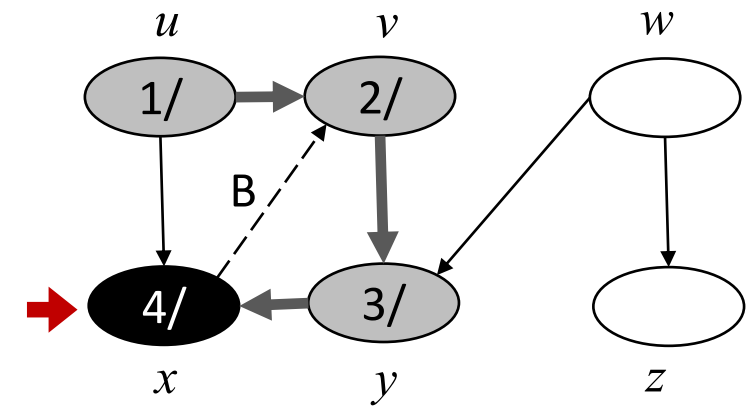
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```

Example



$$G = (V, E)$$

DFS(G)

```

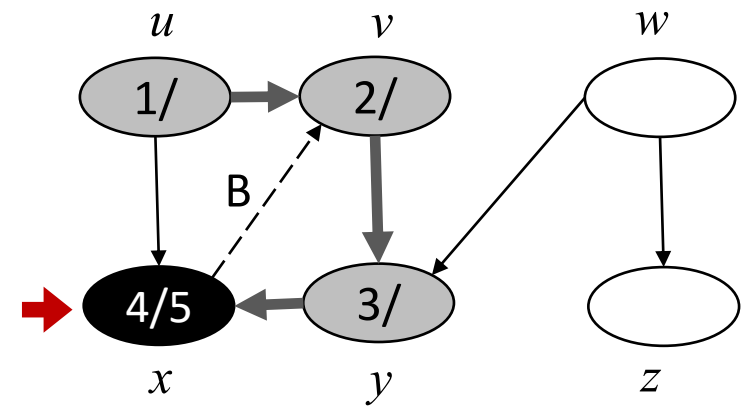
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DFS-VISIT(G, u)

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Example



$$G = (V, E)$$

DFS(G)

```

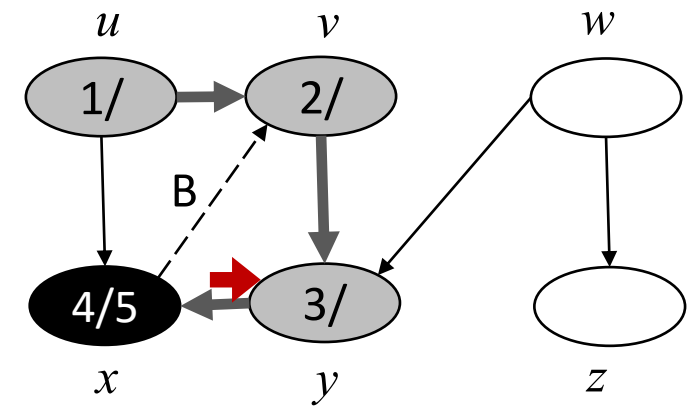
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DFS-VISIT(G, u)

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Example



$$G = (V, E)$$

DFS(G)

```

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```

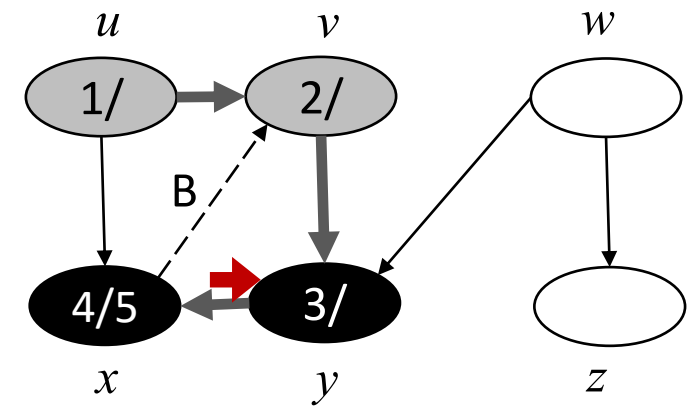
DFS-VISIT(G, u)

```

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```

Example



$$G = (V, E)$$

DFS(G)

```

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```

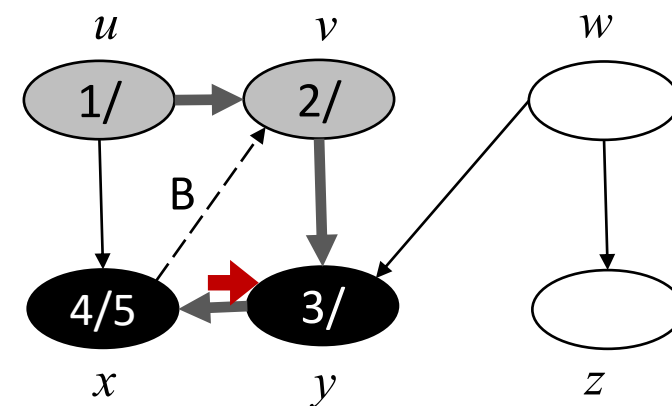
DFS-VISIT(G, u)

```

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3   $u.color = GRAY$ 
4  for each  $v \in Adj[u]$ 
5      if  $v.color == WHITE$ 
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7          DFS-VISIT( $G, v$ )
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```

Example



$$G = (V, E)$$

DFS(G)

```

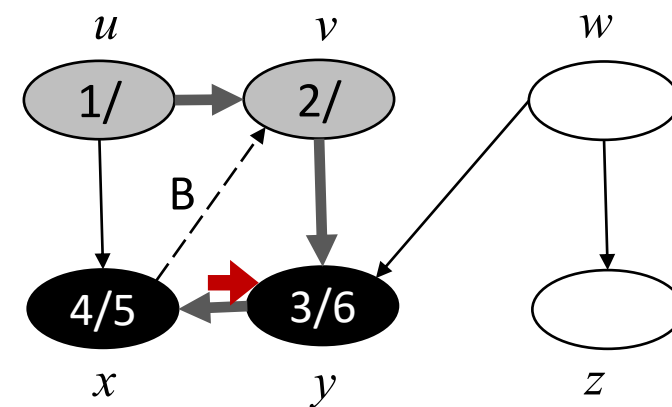
1  for each vertex  $u \in V$ 
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DFS-VISIT(G, u)

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Example



$$G = (V, E)$$

DFS(G)

```

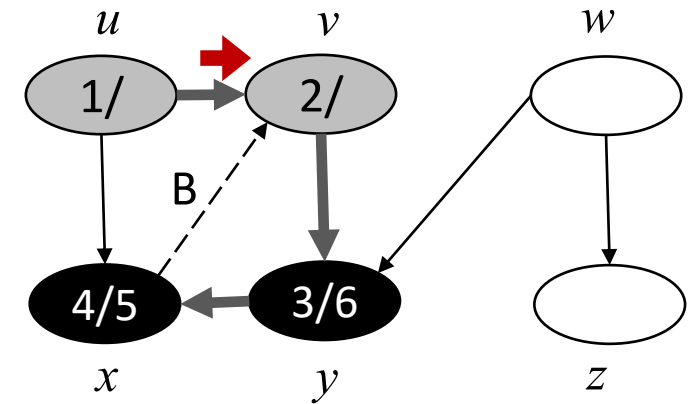
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Example



$$G = (V, E)$$

DFS(G)

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```

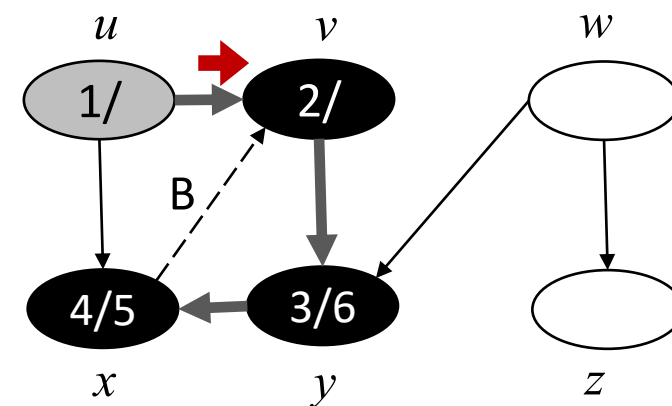
DFS-VISIT(G, u)

```

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7          DFS-VISIT( $G, v$ )
8   $\rightarrow$   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 

```

Example



$$G = (V, E)$$

DFS(G)

```

1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7   $\rightarrow$       DFS-VISIT( $G, u$ )

```

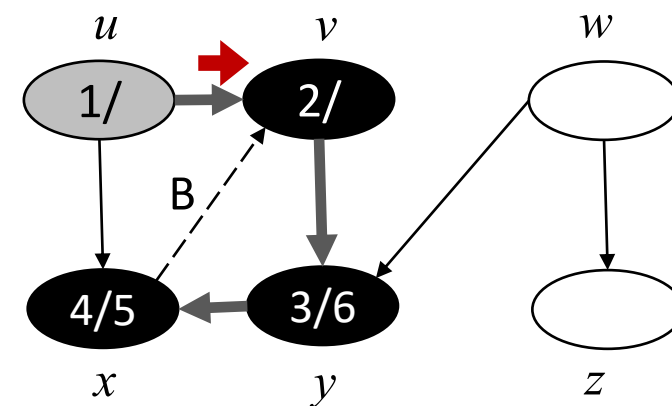
DFS-VISIT(G, u)

```

1   $time = time + 1$ 
2   $u.d = time$ 
3   $u.color = GRAY$ 
4  for each  $v \in Adj[u]$ 
5      if  $v.color == WHITE$ 
6           $v.\pi = u$ 
7          DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
9   $\rightarrow$   $time = time + 1$ 
10  $u.f = time$ 

```

Example



$$G = (V, E)$$

DFS(G)

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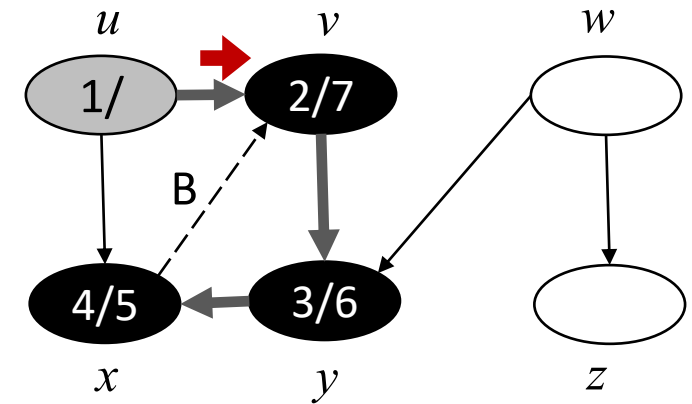
DFS-VISIT(G, u)

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9   $time = time + 1$ 
10  $\rightarrow u.f = time$ 

```

Example



$$G = (V, E)$$

DFS(G)

```

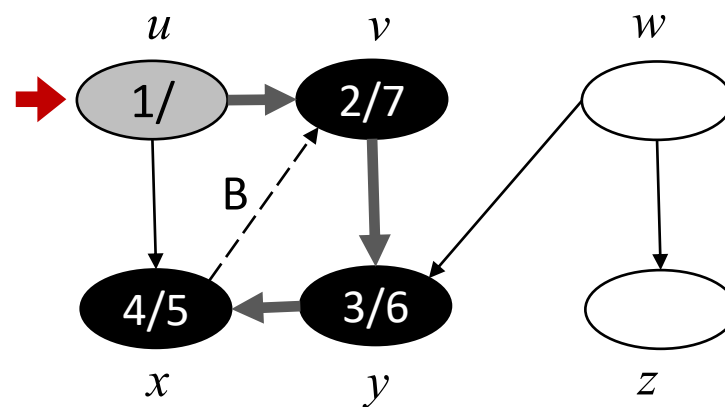
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
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4   $time = 0$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

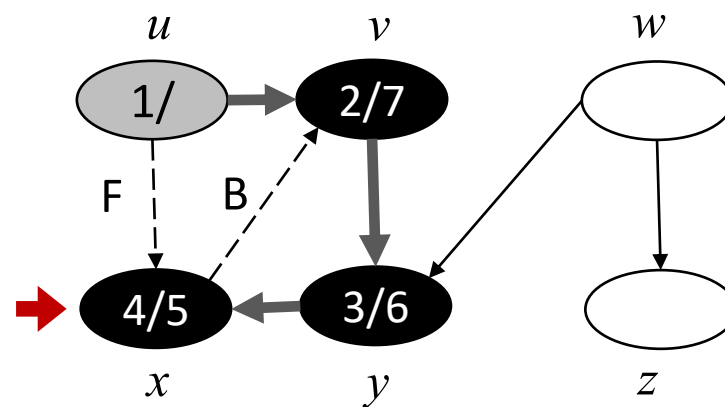
1  for each vertex  $u \in V$ 
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4   $time = 0$ 
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```

DFS-VISIT(G, u)

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8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
    
```

Example



Forward edges are those edges (u,v) connecting a vertex u to a descendent vertex v that is already discovered and finished .

$$G = (V, E)$$

DFS(G)

```

1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7   $\rightarrow$       DFS-VISIT( $G, u$ )

```

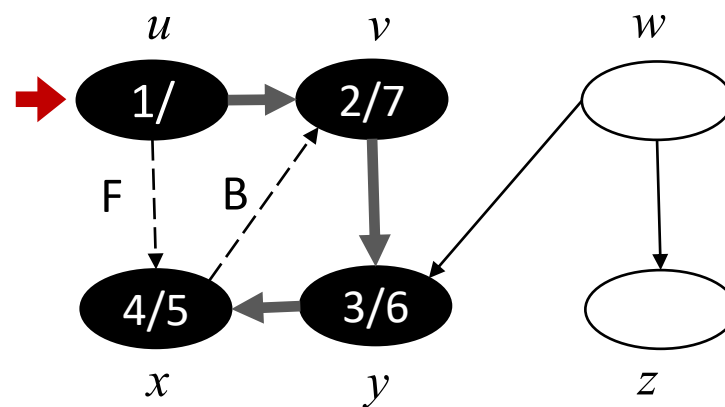
DFS-VISIT(G, u)

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3   $u.color = GRAY$ 
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5      if  $v.color == WHITE$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

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6      if  $u.color == WHITE$ 
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```

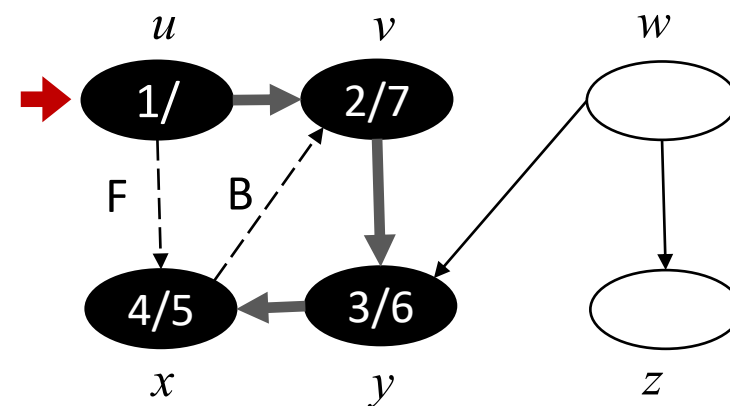
DFS-VISIT(G, u)

```

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2   $u.d = time$ 
3   $u.color = GRAY$ 
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5      if  $v.color == WHITE$ 
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Example



$$G = (V, E)$$

DFS(G)

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```

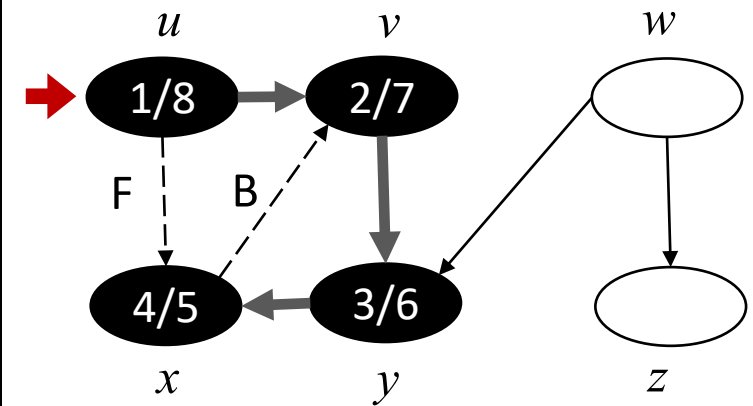
DFS-VISIT(G, u)

```

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2   $u.d = time$ 
3   $u.color = GRAY$ 
4  for each  $v \in Adj[u]$ 
5      if  $v.color == WHITE$ 
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8   $u.color = BLACK$ 
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10  $\rightarrow u.f = time$ 

```

Example



$$G = (V, E)$$

DFS(G)

```

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```

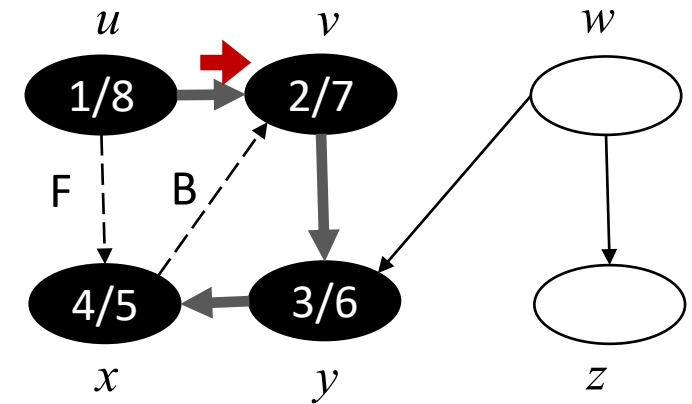
DFS-VISIT(G, u)

```

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3   $u.color = GRAY$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

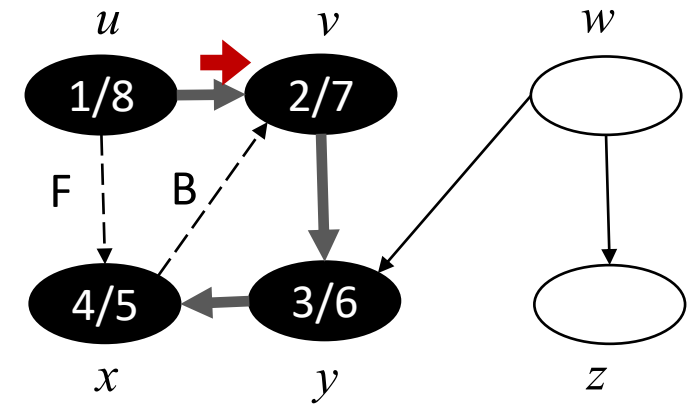
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```

Example



$$G = (V, E)$$

DFS(G)

```

1  for each vertex  $u \in V$ 
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```

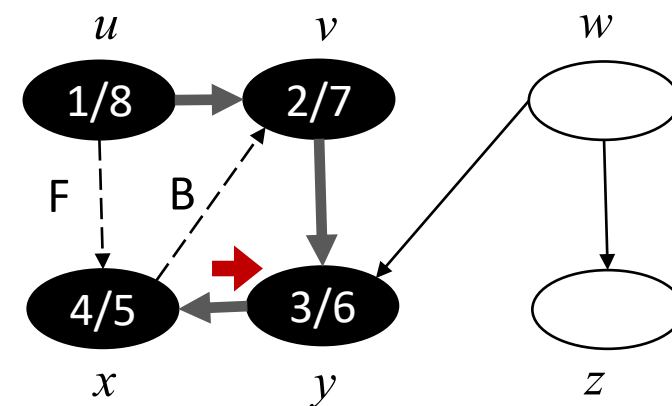
DFS-VISIT(G, u)

```

1   $time = time + 1$ 
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8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 

```

Example



$$G = (V, E)$$

DFS(G)

```

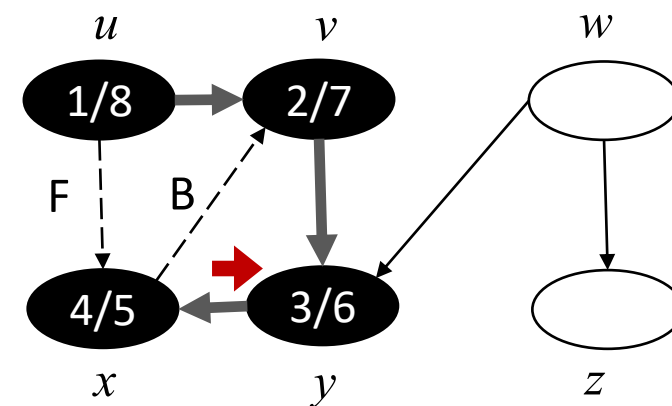
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

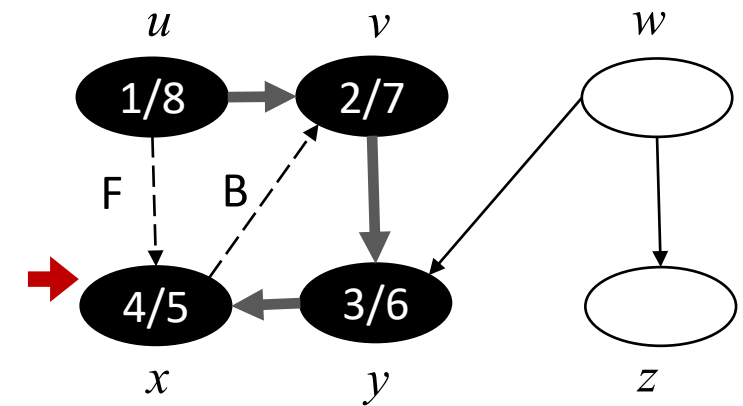
1  for each vertex  $u \in V$ 
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8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
    
```

Example



$$G = (V, E)$$

DFS(G)

```

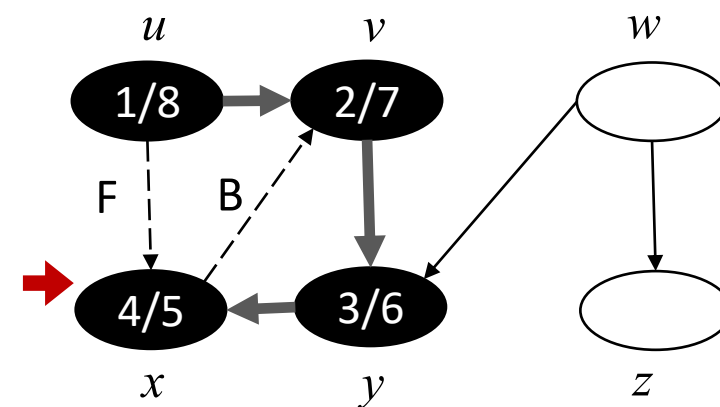
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

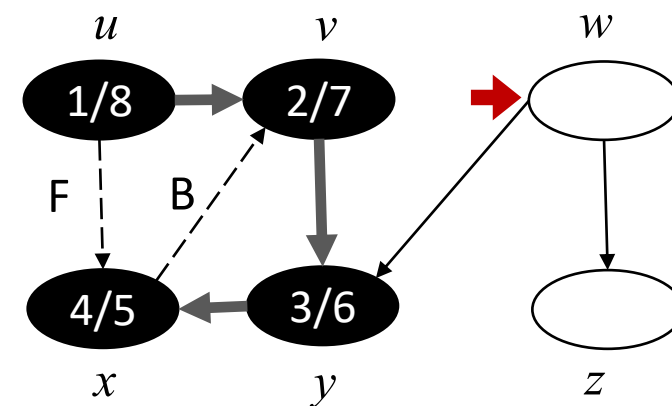
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8   $u.color = BLACK$ 
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10  $u.f = time$ 
    
```

Example



$$G = (V, E)$$

DFS(G)

```

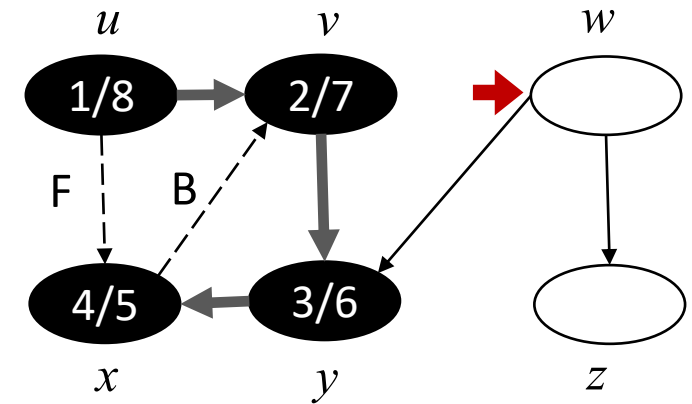
1  for each vertex  $u \in V$ 
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Example



$$G = (V, E)$$

DFS(G)

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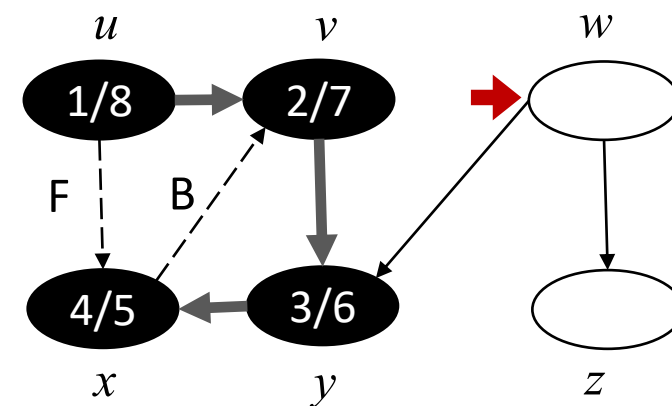
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```

Example



$$G = (V, E)$$

DFS(G)

```

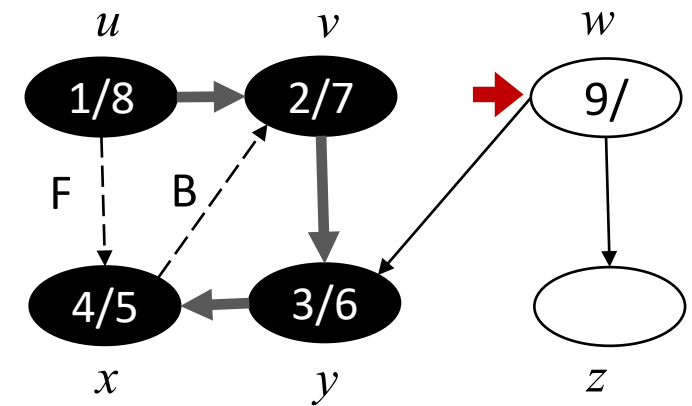
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```

DFS-VISIT(G, u)

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```

Example



$$G = (V, E)$$

DFS(G)

```

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```

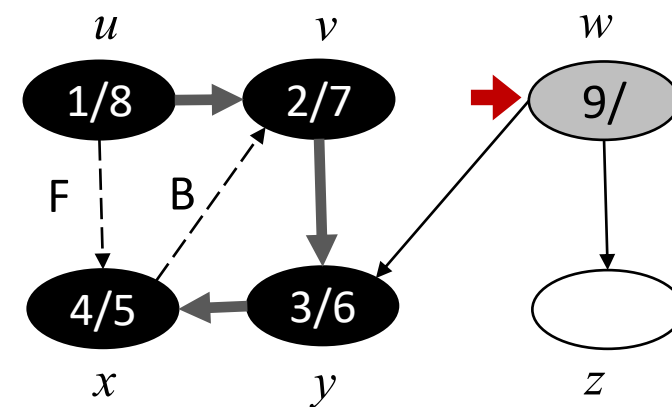
DFS-VISIT(G, u)

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```

Example



$$G = (V, E)$$

DFS(G)

```

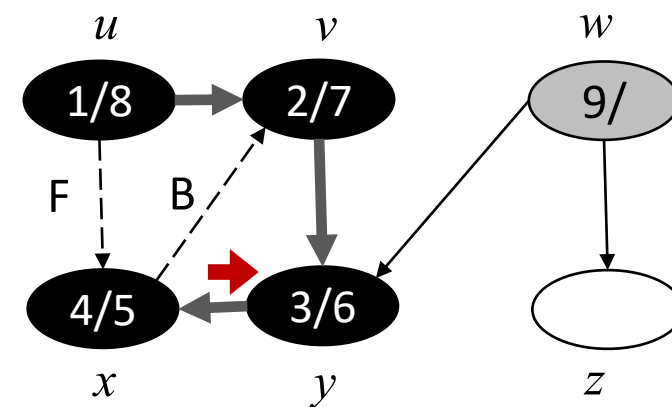
1  for each vertex  $u \in V$ 
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4   $time = 0$ 
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DFS-VISIT(G, u)

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```

Example



$$G = (V, E)$$

DFS(G)

```

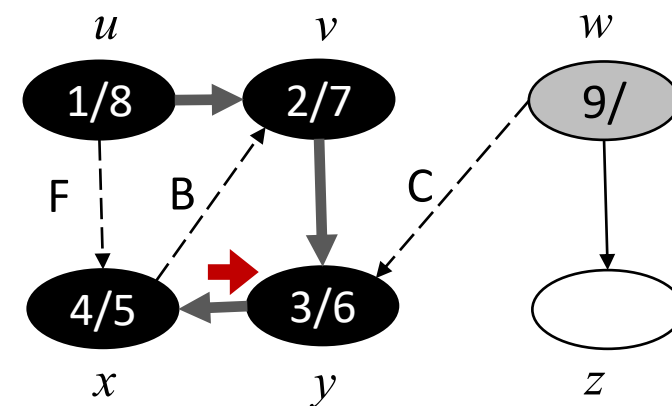
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DFS-VISIT(G, u)

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7              DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
    
```

Example



Cross edges It connects u with already discovered/finished v , but there is no ancestor-descendant relationship between them

$$G = (V, E)$$

DFS(G)

```

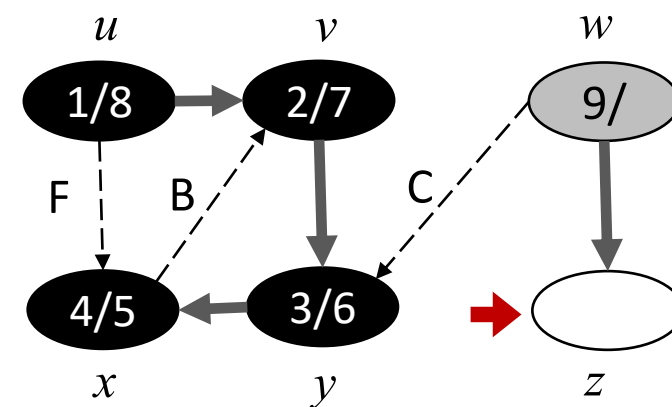
1  for each vertex  $u \in V$ 
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DFS-VISIT(G, u)

```

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8   $u.color = BLACK$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

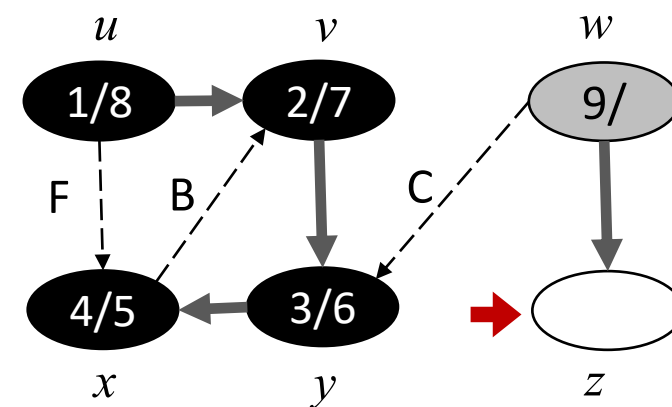
1  for each vertex  $u \in V$ 
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```

DFS-VISIT(G, u)

```

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3   $u.color = GRAY$ 
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7              DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
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10  $u.f = time$ 
    
```

Example



$$G = (V, E)$$

DFS(G)

```

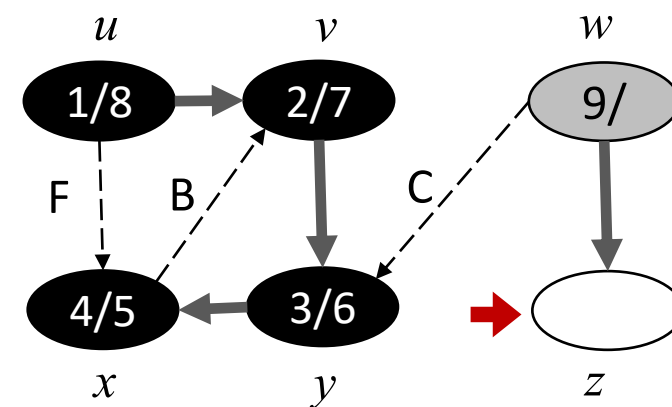
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```

DFS-VISIT(G, u)

```

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7          DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
  
```

Example



$$G = (V, E)$$

DFS(G)

```

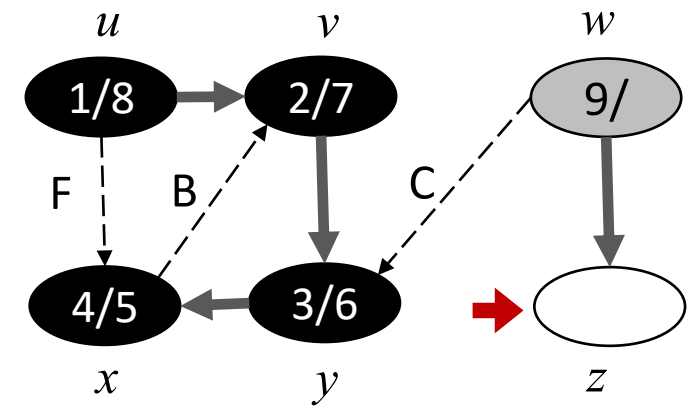
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7   $\rightarrow$       DFS-VISIT( $G, u$ )
  
```

DFS-VISIT(G, u)

```

1   $time = time + 1$ 
2   $u.d = time$ 
3   $u.color = GRAY$ 
4  for each  $v \in Adj[u]$ 
5      if  $v.color == WHITE$ 
6   $\rightarrow$        $v.\pi = u$ 
7          DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
9   $time = time + 1$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

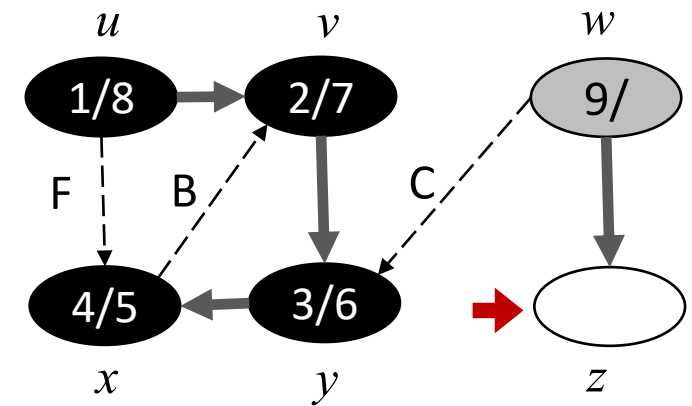
1  for each vertex  $u \in V$ 
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5  for each vertex  $u \in V$ 
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8   $u.color = BLACK$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

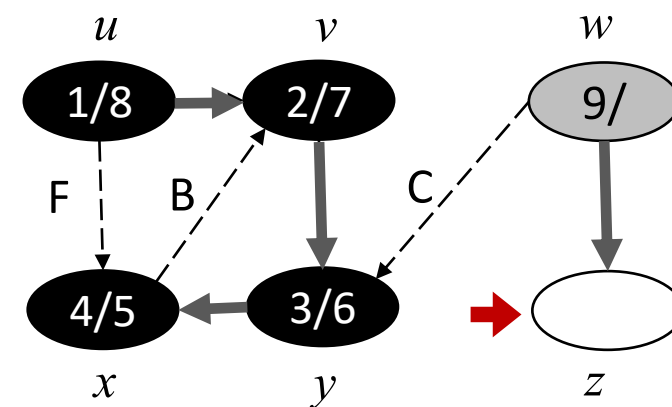
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7   $\rightarrow$       DFS-VISIT( $G, u$ )
  
```

DFS-VISIT(G, u)

```

1   $\rightarrow$   $time = time + 1$ 
2   $u.d = time$ 
3   $u.color = GRAY$ 
4  for each  $v \in Adj[u]$ 
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8   $u.color = BLACK$ 
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```

Example



$$G = (V, E)$$

DFS(G)

```

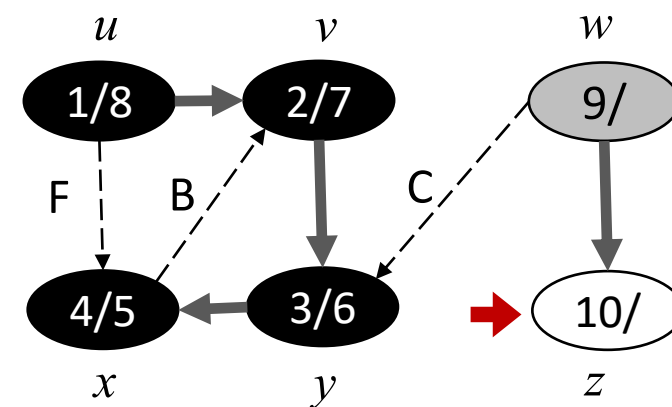
1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
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```

DFS-VISIT(G, u)

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```

Example



$$G = (V, E)$$

DFS(G)

```

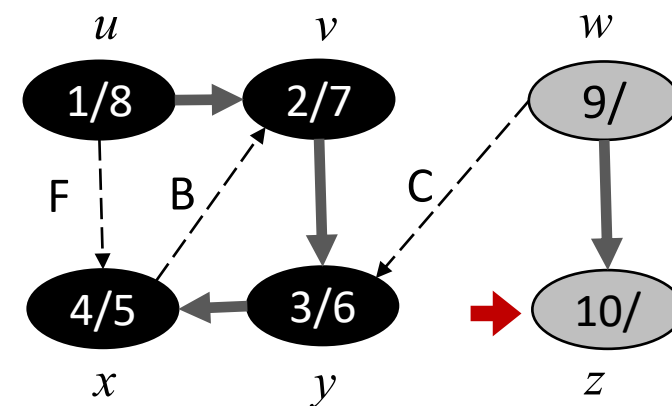
1  for each vertex  $u \in V$ 
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```

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```

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```

Example



$$G = (V, E)$$

DFS(G)

```

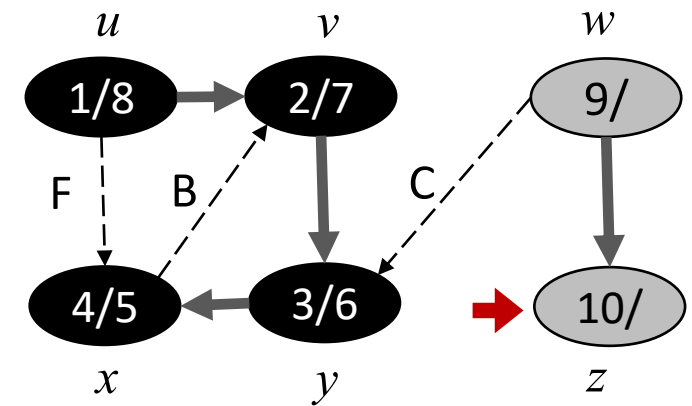
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Example



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DFS(G)

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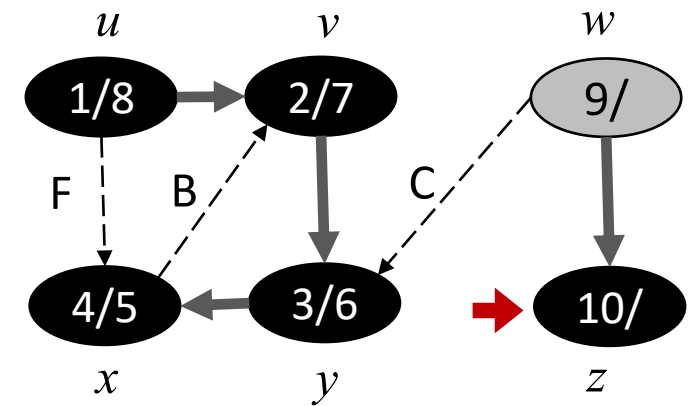
DFS-VISIT(G, u)

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Example



$$G = (V, E)$$

DFS(G)

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```

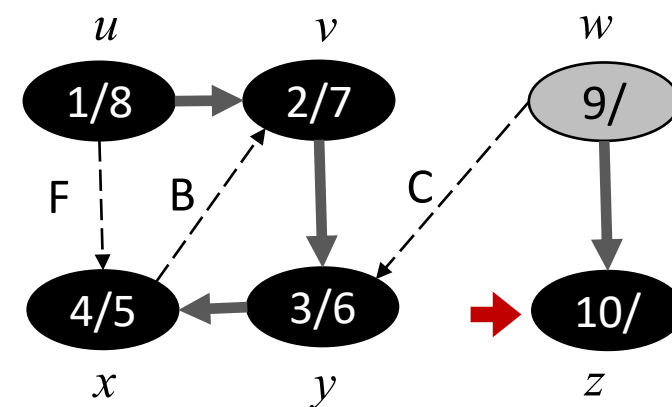
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Example



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DFS(G)

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```

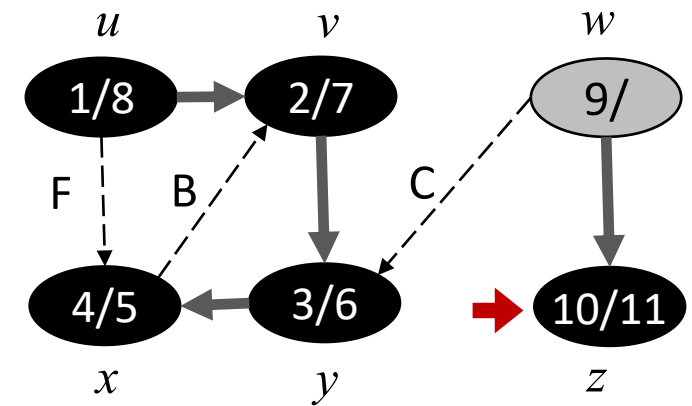
DFS-VISIT(G, u)

```

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```

Example



$$G = (V, E)$$

DFS(G)

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7  →      DFS-VISIT( $G, u$ )

```

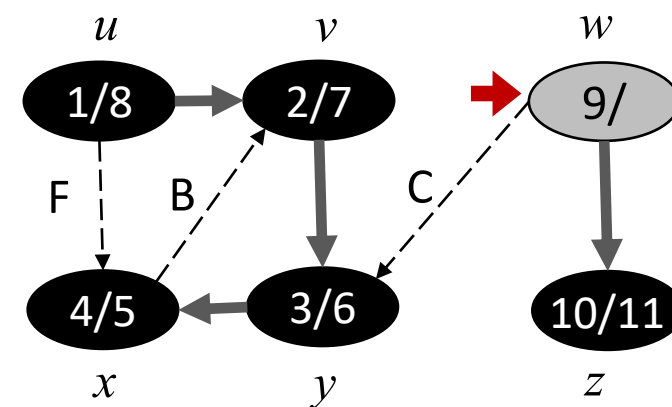
DFS-VISIT(G, u)

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Example



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DFS(G)

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```

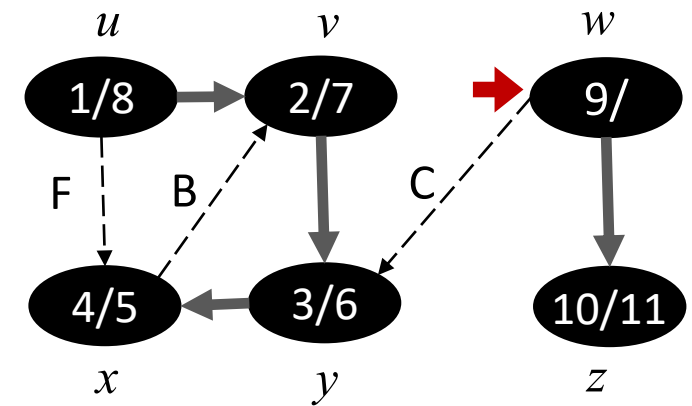
DFS-VISIT(G, u)

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Example



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DFS(G)

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```

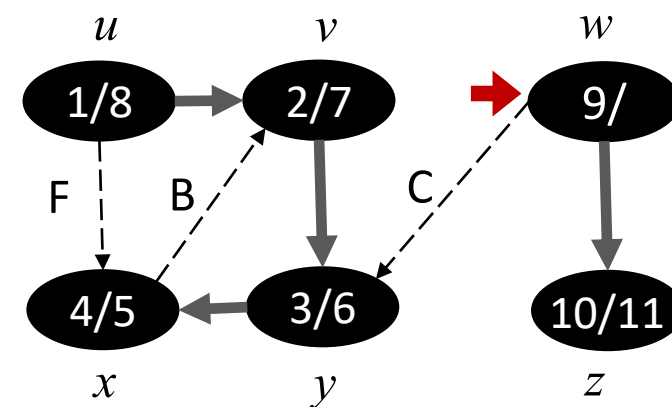
DFS-VISIT(G, u)

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Example



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DFS(G)

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```

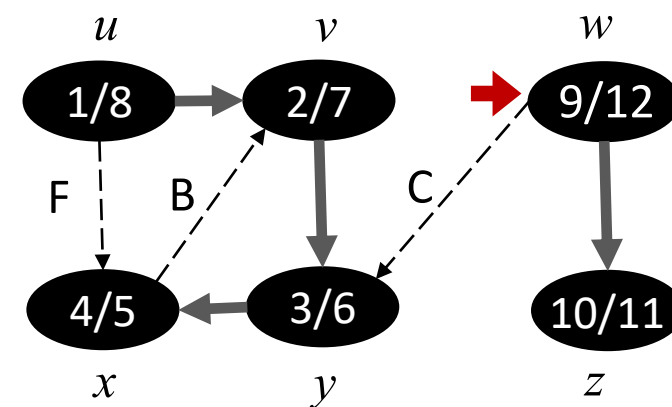
DFS-VISIT(G, u)

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```

Example



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DFS(G)

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```

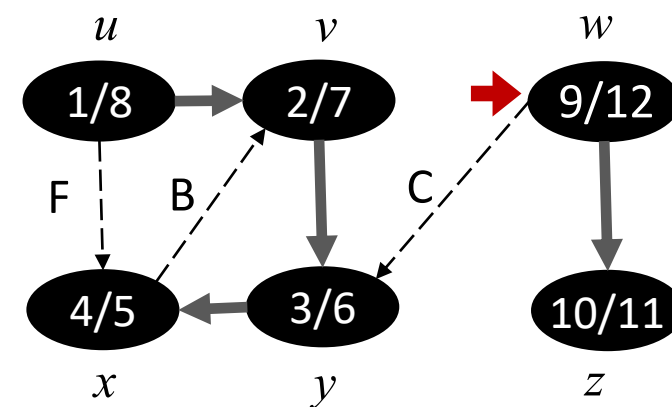
DFS-VISIT(G, u)

```

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Example



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DFS(G)

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```



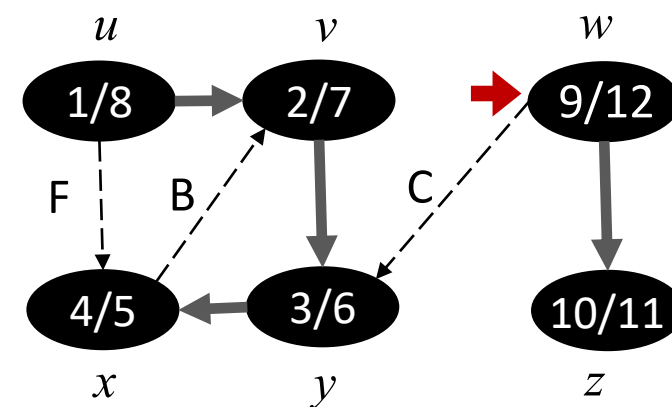
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8   $u.color = BLACK$ 
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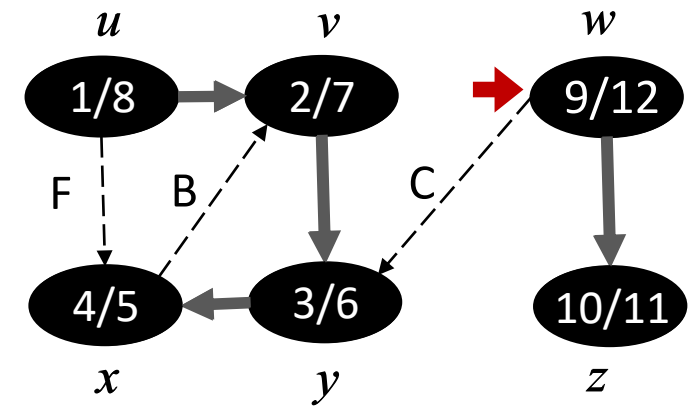
```

Example



- DFS creates a forest $G_\pi = (V, E_\pi)$, a collection of **rooted trees**, where
 - $E_\pi = \{(v.\pi, v) : v \in V \text{ and } v.\pi \neq \text{NIL}\}$
 - All the edges in E_π are tree edges

Example



$$G_\pi = (V, E_\pi)$$

$$V = \{u, v, w, x, y, z\}$$

$$E_\pi = \{(u, v), (v, y), (y, x), (w, z)\}$$

DFS Running time Analysis

$$G = (V, E)$$

DFS(G)

```

1  for each vertex  $u \in V$ 
2       $u.color = WHITE$ 
3       $u.\pi = NIL$ 
4   $time = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.color == WHITE$ 
7          DFS-VISIT( $G, u$ )
    
```

} V

} V

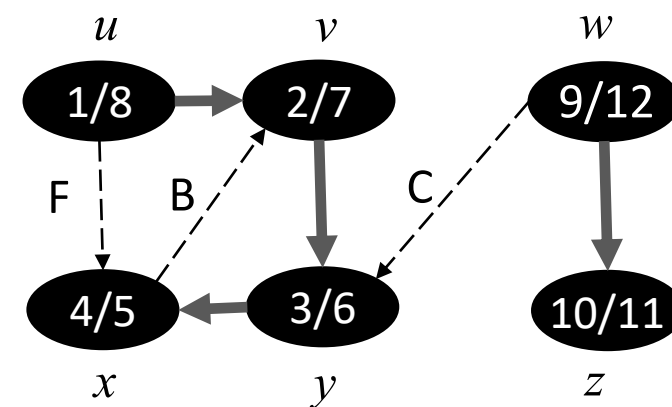
DFS-VISIT(G, u)

```

1   $time = time + 1$ 
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8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
    
```

} $Deg(v)$

Example



DFS Running time Analysis

$$G = (V, E)$$

DFS(G)

```

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6      if  $u.color == WHITE$ 
7          DFS-VISIT( $G, u$ )
    
```

$\left. \begin{array}{l} \text{lines 1-3} \\ \text{lines 5-7} \end{array} \right\} V$

DFS-VISIT(G, u)

```

1   $time = time + 1$ 
2   $u.d = time$ 
3   $u.color = GRAY$ 
4  for each  $v \in Adj[u]$ 
5      if  $v.color == WHITE$ 
6           $v.\pi = u$ 
7          DFS-VISIT( $G, v$ )
8   $u.color = BLACK$ 
9   $time = time + 1$ 
10  $u.f = time$ 
    
```

$\left. \begin{array}{l} \text{lines 4-7} \\ \text{line 8} \end{array} \right\} \text{Deg}(u)$

- Loops on lines 1-3 and lines 5-7 take time $\Theta(V)$, exclusive of the time to execute the calls to DFS-VISIT
- During an execution of DFS-VISIT(G, v), the loop on lines 4-7 executes $|Adj[v]|$ times.
- Since $\sum_{v \in V} |Adj[v]| = \Theta(E)$, the total cost of executing lines 4-7 of DFS-VISIT is $\Theta(E)$
- Therefore, the running time of DFS is $\Theta(|V| + |E|)$

Thanks a lot



If you are taking a Nap, **wake up**.....Lecture Over